JOINT SERVICES
COMMAND AND STAFF COLLEGE

DEFENCE RESEARCH PAPER

By

MAJ H P MANSIKKA

ADVANCED COMMAND AND STAFF COURSE

NUMBER 10

OCT 06 - JUL 07

Can the military intuition be enhanced by game theory?

By

Major Heikki Mansikka

Word Count: 16000

Abstract

The aim of this paper is to describe how a game theory can be used to enhance military intuition in a typical military decision making process. The paper seeks to explain the essential theoretical principles of the game theory and continues by showing how those principles could be transitioned into a dynamic game.

Game theory is an effective tool to structuralize complex decision making problem that includes multiple actors. Also, a game theory helps the decision maker to understand the risks included in the dynamics of the game and to comprehend what actually can and cannot be achieved in the given game situation.

The outcomes of military decision making processes are widely known as they generate acts that are well covered in the modern media. The actual process through which the military decisions are being generated is perhaps less understood. In the military decision making process, there are elements that require logical skills and elements which are more based on intellectual genius or intuition. Due to the mass of information related to and the complexity of a typical military planning problem, even the intuition part of the process is generally being supported by different kind of analytical tools and simulations. Those tools are typically under the control of specialists and the logic of those tools is often left rather mysterious even for the military planners.

The purpose of this paper is to describe how a game theory can be used to enhance military intuition in a typical military decision making process. The paper seeks to explain the essential theoretical principles of the game theory and continues by showing how those principles could be transitioned into a dynamic game model. The aim is neither to explain all the elements of the game theory nor to construct and solve a complete game. Instead this paper seeks to build an example that is sufficient for the understanding of the general game theoretic principles and detailed enough for the critical evaluation of the constructed game model.

This paper starts by describing an imaginary but typical military decision making situation and discusses very briefly how the problem solving is done in a doctrinally pure manner in the armed services of the United Kingdom.

That is followed by an introduction to game theory. This section explains the key elements and principles of the game theory using a step by step approach. The section starts from the basic definitions and premises and continues into some more complex implications of the game theory. The main purposes of this section is first to introduce the main principles of the game theory to a reader who is not familiar with the subject and second, to lay down the theoretical foundation for the game model that is introduced in the section that follows. Only those aspects of the game theory are being explained that are considered necessary for the understanding of the following demonstration. Those familiar with the game theory may find this section somewhat lengthy and narrative. But is has to be elaborated that the paper assumes no previous knowledge of the game theory from the reader. Finally, the section creates a foundation against which the accuracy and truthfulness of the game model, which forms the core of the paper's intellectual body, can be evaluated. In order to help the orientation to the formulated game, the section's essential elements are explained in examples which are at least loosely related to the principles of the model that follows. Throughout the section the approach is kept as non-mathematical as practically possible.

Once the necessary theoretic understanding is confirmed, the paper proceeds by formulating a game theoretical solution for the military problem described in the previous section. The formulation of the game is again done in steps as the subject is somewhat complex. The game is based mostly on the principles described in the preceding section. Still, there are some game specific solutions that are introduced here for the first time. This is done for the reason that if those principles would have been explained earlier the boundaries of the general theory and the game unique problems would have become blurred and difficult to follow. The first key element of the game is the game tree around which the whole logic of the game is constructed. Only examples that are necessary for the understanding of the logic of the game tree are being included in the paper. The second key element of the game is the formulas used to define players' successes during the game. All used formulas are being described in detail but the actual mathematical calculations are mainly being excluded as unnecessary. The game is solved on example basis only but at the same time in a way that enables anyone to complete the whole game using the provided information.

As a conclusion this paper finishes by drawing together the key elements of the research and by summarising the main findings of the previous sections. The main emphasis is put on the analysis of the constructed game model. The section concentrates on highlighting the strengths and weaknesses of the model and presents some findings and suggestions for further efforts to enhance the model.

The main arguments of this paper are as follows. A game theory is an area of applied mathematics that can, at least to some extent, be understood with common sense and average intelligence and without understanding of the higher level mathematics. The game theory is an effective tool to structuralize complex decision making problem that includes multiple actors. Also, a game theory helps the decision maker to understand the risks embedded in the dynamics of the game and to comprehend what actually can and cannot be achieved in the given game situation.

Naturally the game theory comes with some limitations. First of all, if the game situation is very complex the formulation of the game could turn out to be an extremely challenging task. Once the game tree is structured the players' behaviours need to be embedded in to the game's dynamics. An essential premise of the game theory is that all players behave rationally. This assumes that the players can mentally 'play the game forward' as the game proceeds thereby making selections that maximise their individual benefits. Also, depending on the structure of the game the players are more or less capable of seeing other players' decisions and decisions made earlier in the game. None of these principles fit well in the nature of war where each actor generally tries to cover their actual decisions if it is considered beneficial. In addition, those waging war are often forced to operate with very limited understanding of the surrounding reality. In this sense the simple games need to be understood as modelling tools only, which can not foretell the actual outcome of a complex military campaign. In a game theory there are methods for capturing even the most complex elements of game dynamics, such as the limited information and cheating. But as this paper has a non-mathematical approach, most of those mechanics are being left untouched.

One of the biggest challenges when constructing a game is the process of generating the formulas that define the game's outcomes and affect the player's behaviour. It is left for the researcher's intellectual decision to define the different weights within the formulas. The given weights reflect player's preferences and directly affect the game's outcome. Unfortunately many games, including the one formulated in this paper, has elements that are difficult to weight.

Games often include so called nature or chance nodes whose outcome is solved using some software based simulation tool. Even though the simulations and difficulties related to them fall outside the context of the actual game theory, some emphasis on simulation is necessary in order to understand the problems related to game theoretic modelling. In this paper the ACEM (Air Combat Evaluation Simulation Model) software was used to solve the nature nodes. This was essential as otherwise even a partial solution of the game would had been impossible. Whereas a theoretic model of the game includes numerous user defined variables and weights, the sophisticated simulation programs are in that sense in a league of their own. Too often it is forgotten that simulation programs are based on user defined behaviour rules and that those rules can be modified freely. Basically, with the use of game theoretic model and simulation tool it is possible to come up with any kind of results wanted. But if the simulation tools and the game theory are being used unethically or with biased focus, it is not the fault of either the game theory or the simulation tool.

The operational military planning in the United Kingdom's armed forces is based on a process called an estimate. The estimate is an analytic and systematic process used for problem solving and decision making. Inside the estimate process there are various methods and procedures that help decision makers to conceptualise the given problem. Despite the rather controlled and formal process it is the traditional military intuition that is still largely required at the key stages of the process.[1] Without discussing the estimate process in any detail it is essential to broadly elaborate some elements of it. As the planning staff plans an operation, they usually produce several optional courses of action (COA) for the Commander to choose from. It is at this point when the intuition is at its peak. Commander is probably given assumptions about the pros and cons of each course of action. Commander may even be given a numeric presumption of the likely outcomes of different options. If the estimate process is timely compressed or there are no operational analysts available, the reliable numerical data is unlikely to be available. But comparing numbers is generally easier than comparing completely verbal values. And in military environment the

---

[1] Ministry of Defence, *The Joint Doctrine and Concepts Center, Joint Warfare Publication 5-00 Joint Operations Planning* (Ministry of Defence, 2004), Annex A.

numbers matter whether they are the number of casualties suffered, number of civilians getting killed, miles of land cleared or a share of domestic audience favouring the military campaign. However one puts it, the military decision maker ultimately faces the world of numbers, whether they were explicitly given to him or not.

This paper takes an imaginary and extremely simplified military decision making problem and tries to found a numerical answer for it by using game theory. The paper assumes that a Commander is given several optional courses of action to choose from. He has no support from the analysts and the options are verbal only. A Commander, concerned about those measurable values just mentioned, asks; *'If the enemy is able to allocate four air defence fighters to the area 'XX', what is the expected outcome against target 'YY' when four friendly air superiority fighters and an air interdiction unit is used?*' In this case the commander would be left without proper answer as anything but a numerical answer would be probably considered unsatisfactory. This paper studies if the Commander's intuitive decision could be supported by game theory. For the Commander's question to be answered more satisfactory an understanding of the principles of the game theory is required. Before this paper moves to applying game theoretic models and simulation into the military decision making process some basic principles of the game theory shall be clarified and explained. The following section explains the essential elements, premises, principles and definitions of the game theory.

A game theory belongs to an area of applied mathematics. With a game theory it is possible to study and model conflicts and interactions of multiple actors. A typical game theoretic study is done by using mathematical game models.[2] At its simplest, the game theory explains the logical interactions between a minimum of two actors in a situation where they may have opposing interests. The opposing interests mean that there exists a conflict between the actors.

The game theoretic models are normative by nature. Therefore, based on their findings they are not attempting to predict how the real players would act in a conflicting situation. Instead, the game theoretic models are describing and studying how the rational actors would act in a precisely defined environment. For this reason the game theoretic models should be understood as approximations of the real-life interactions.[3]

In a game, there are always at least two actors who are able to make decisions affecting to each other. If there is only one actor or actors' decisions do not affect each other there is no interaction that could be defined in terms of game theory. Instead, the process is then more about making a decision and falls into the area of a decision making analysis. The decision analysis is very briefly

---

[2] Poropudas, Jirka, *Ilmataistelun simulointimallin peliteoreettinen analyysi* (Helsinki: University of Technology,System Analysis Laboratory, 2005), research paper, p. (introduction).
[3] ibid.

discussed later in this paper. The game theory defines the interacting actors as players. In a n-player game the players are numbered 1….n. The arbitrary player is described with a symbol i. The players in a game are therefore expressed in a following manner; Pi, i= 1…n. [4] The players may cooperate or they may have opposing interests. It is also possible to formulate a game where there are two groups of players. The two groups can have opposing interests but within the group the group's players can have shared interests.[5] This is the formulation that is being used in this paper's game theoretic case study.

The following analysis uses terms 'strategy' and 'tactics' with exclusive meanings and these terms are not consistent with their meanings in traditional military context. For the purposes of this paper, the strategy is defined as the sum of the player's decisions within a game or a sub-game.[6] Tactics is defined as player's decision made in a single decision node.

The basic assumption is that the players are free to select their behaviour. All the possible strategies the player has at hand form a group of strategy choices. This group of choices is called a player's strategy group or player's strategy space.[7] The player's strategy space is expressed with a symbol Si. The arbitrary strategy within Si is expressed with a symbol si. The strategy space can be therefore expressed in a form; Si, i=1…n. Furthermore, the fact that si belongs to a strategy space Si is expressed in a form; si ε S1.[8] Once the outcomes of all possible strategy combinations have been calculated, it is possible to search for the so called warranty strategies. The warranty strategy means that by playing that strategy combination the player gets an optimal outcome despite the strategies the other players might select from their strategy spaces.[9] When a player wants to minimize his losses he can utilize the warranty strategy thereby minimizing the negations of his expected utility. By doing so, the player selects logic of; 'By selecting this strategy I can not lose more than that defined by this strategy. And I do my choices regardless of strategies the other players may or may not select'. On the other hand, when a player wants to maximise his benefits the warranty strategy maximizes his possible gain regardless of the strategy selections the other players might do. A game where one player tries to maximize his benefits while the other player is minimizing his losses is commonly called a Minimax or Maximin game.[10] A player may wish to randomise his decisions between two or more different strategies. Such a strategy combination is

---

[4] Gibbons, R, *A Primer in Game Theory*, (Harlow, Essex: Prentice Hall,1992), p. 3.
[5] Rasmussen, E., *Games and Information: An Introduction to Game Theory* (Oxford, Blackwell Publishers,2001), p. 29.
[6] Shubik, Martin, *Readings in Game Theory and Political Behavior* (Garden City, New York: Doubleday, 1954) p. 34.
[7] Poropudas, op.cit.
[8] Gibbons, op.cit., p.3.
[9] Karelahti, Janne, Untitled, (Helsinki: University of Technology, System Analysis Laboratory, 2006), unpublished lecture pamphlet, no page numbering.
[10] LtCol Cantwell L., Gregory*, Can two person zero sum game theory improve military decision making course of action selection?,* (Fort Leavenworth, Kansas: School of Advanced Military Studies, United States Army Command and General Staff College, 2003), A Monograph, p. 10.

called a mixed strategy.[11] There are always probabilities included when a player makes a decision to either select or not to select a certain strategy. A mixed strategy emphasises the player's uncertainty about the most beneficial decision.

Assuming that all players are making rational decisions it is possible to search for the game's equilibrium or saddle point. Saddle point defines a strategy combination that always results in a highest possible utility. The further away the player drifts from this saddle point less is his utility once the game comes to an end. [12] The game's saddle point or equilibrium can be different depending on whether the player uses mixed strategies or non-mixed, pure strategies i.e. strategies which are not randomised. There are many cases where a player is able to maximise his utility only by using mixed strategies.[13] A simple example of this follows. In a two player game there is a player B (Blue) who is able to launch his air offensive against one city and is able to select his target between cities x, y or z. From the blue player's perspective all cities provide equal benefits and attack against any city involves an equal risk. At the same time a player R (Red) only has enough resources to allocate air defence fighters to protect either city x, y or z respectively. The assumption is that if the city is undefended it will be totally ruined by player B's bombers. Also, if the city is defended by player R's air defence fighters the city will be completely saved. From the player R's point of view the B's target selection would seem as a random process. But if the player B would always attack against city x, it would be fair to argue that the B's decision would no longer be random and the cities would be less likely to be hit successfully. The latter includes an assumption that the player R would believe the player B's stubborn attitude of avoiding mixed strategies.

Payoff is a term widely used in game theory. Payoff is related to game models where the term means the player's resultant outcome of the game.[14] Payoff is usually defined in terms of money or other such unit that has a natural scale.[15] If there is no such natural scale related to the payoff it is possible and also necessary to create a scale in order to be able to rate the payoffs.[16] For example, in an air combat a loss of an aircraft can be rated with a certain value. Whether this scale is reflecting the true value or not is a critical issue as a misleading scale will lead to compromised modelling results. Players are expected to choose their strategies in a manner that maximises their outcome. This means that it is assumed that players act rationally and selfishly. The players' rationality is an essential premise in all game theoretic models. It is important to

---

[11] Carmichael, Fiona, *A Guide to Game Theory*, (Harlow, Essex: Prentice Hall, 2005), p. 13.

[12] LtCol Cantwell L., Gregory, op.cit. p.9.

[13] Rapoport, Anatol, *Fights, Games, and Debates* (Ann Arbor, Michigan: University of Michigan Press, 1960) p.156.

[14] Coram, Alex, Talbot, *State, Anarchy and Collective Decisions* (New York: Palgrave, 2000), p. 14.

[15] Budescu, David V., Erev, Ido, and Zwick, Rami, eds., *Games and Human Behavior: Essays in Honor of Amnon Rapoport* (Mahwah, NJ: Lawrence Erlbaum Associates, 1999) p.390, [online], Available from: http://www.questia.com/PM.qst?a=o&d=26304023, [Accessed 2 May 2007].

[16] Clemen, Robert T, *Making Hard Decisions, An Introduction to Decision Analysis*, (Duke: Duxbury Press,1996), p.128-135.

remember that the players' rationality is not always self evident. Some action that seems irrational to other could be rational to other. This is true even when there is a natural scale to measure the absolute payoffs. This means that there may be other unidentified, cultural or religious values that may reverse the observed rationality.

Utility (u) is another essential game theoretic concept. Utility is the player's subjective view of the outcome's value. The utility includes not only measurable values but also player's preferences. In a game there is a specified utility related to each strategy a player might choose. Utility is usually described as an utility function and is written in a form; $ui\ (s1\ldots\ldots sn)$, $si\varepsilon\ S1$.[17] The utility function originates from the utility theory which is one branch of the decision theory. [18] The utility function is used in decision models where uncertainties are involved. If the decision model is deterministic the utility function is replaced with a value function.[19] In game models it is possible to use the utility function instead of the payoff. The game theoretic model of this paper uses the utility function. If players know the utility functions of other players it allows them to execute rather straightforward analysis about the outcomes of different strategy combinations. So called complete information or common knowledge assumes that all the players are aware of the other players' utility functions.[20] Incomplete information on the other hand assumes that players do not know the utility functions of other players.[21] In reality it is not quite that easy to gain knowledge of the other players' utility functions. In an air combat one could be able to define players' utilities in terms of victories and losses i.e. exchange ratios. Even if that information would be available, it would be extremely difficult to dictate players' preferences towards those true or predicted victories and losses. This is not a problem uniquely related to game theory as such preference related value calculations are common to most if not all military decision making problems seeking for optimum effect. But it should be fair to argue that when military decision making is supported by game theoretic models, the relevance of correct value assumptions is highlighted.

Perfect information assumes that when a player is making his decision he knows how the game has evolved to that point.[22] Therefore, with perfect information the player knows what has happened in the game so far and what decisions other players have made.[23] A game of perfect information is a rather uncommon phenomenon in an armed conflict where great emphasis is put on operation security and deception. Still, in an air warfare context, a classical visual combat could be considered as a game of perfect information. There both players typically see each others constantly and therefore know the selected strategies and strategy combinations resulting to a

---

[17] Karelahti, op.cit.
[18] Clemen, Robert T, op.cit., p. 530-537
[19] Karelahti, op.cit..
[20] Gibbons, op.cit. p. 55
[21] Karelahti, op.cit.
[22] Kelly ,Anthony, ed., *Decision Making Using Game Theory: An Introduction for Managers* (New York: Cambridge University Press, 2003) p. 5.
[23] Karelahti, op.cit..

certain stage in a fight. When the aerial combat extends beyond the visual range arena the situation is quite different and approaches the situation most likely encountered in the modern armed conflict. There players collect observations about surrounding reality based on their technical capabilities and personal ability. This information gathering process, also known as situation awareness build-up, is seldom efficient enough to enable players' to capture all the relevant events of the game or the battle. It is also possible that only one player or group of players have perfect information. In this case the other player receives no or limited information of the strategies used.

A strategy is said to be dominant when that strategy maximises the player's payoff despite the strategy selections the other players might take.[24] The dominance can be either strong or weak. The dominance is considered to be strong if the strategy maximises player's payoff against any strategy selected by the opponent. The dominance is said to be weak if a player selecting the weakly dominant strategy is able to achieve at least the same payoff as he would do by selecting some other strategy.[25] It is expected that a rational player will not play a strategy that is being dominated. [26] In a game theory the dominance is expressed in the following manner;

For a player A the strategy A_i is a dominant strategy if (A_{-i}, B_{-i}):

$P(A_i, B_i) > P(A_{-i}, B_i)$ ja $P(A_i, B-_i) > P(A_{-i}, B-_i)$, where

(i) $P(A_i, B_i)$ is player A's payoff when he selects strategy A_i and the player B selects strategy B_i.
(ii) $P(A_{-i}, B_i)$ is player A's payoff when he selects any other strategy than A_i and the player B selects strategy B_i.
(iii) P(Ai, B-i) is player A's payoff when he selects strategy Ai and player B selects any other strategy than Bi.[27]

The strong and weak dominance is possible to be defined also in games where players randomize their strategies over some level of probability i.e. players use mixed strategies.[28] The dominance is not granted in modern warfare. Western powers often pose superior capabilities related to information and other key aspects of military power. Still they are repeatedly undermined by opponents who seem to be dominated in all areas required for decisive victory. If the game theoretic models are extended to strategic and political levels they are unlikely to be able to capture the essential nature of players' capabilities and strategy choices. This view is supported by

---

[24] Carmichael, op.cit. p. 22
[25] Carmichael, p. 25
[26] Karelahti, op.cit.
[27] Carmichael, op.cit. p.38.
[28] Karelahti, op.cit.

the asymmetric and manoeuvrist nature of most modern military threats. These threats, if successful, are able to do the unexpected therefore diminishing the value of pre-calculations and assumptions of available strategies and existing dominances. In a way, these threats select strategies that are not recognised as possible.

When a military decision making is supported by the game theory, one has to understand what type of game best captures the nature of the problem at hand. Different game situations can be roughly divided into normal form games and expanded form games. The normal form game is often called a simultaneous move or a static game. The expanded form game is also known as a dynamic game or a sequential move game. In a static game the players choose their strategies at the same time. It is important to note that this does not necessarily mean that the players would act simultaneously.[29] Based on what has been stated earlier the static or normal form game (G) can be expressed as follows; $G= \{S_1.....S_n; u_1.....u_n\}$.

When a military problem is defined as a game one has to dictate what is the relation between the players' payoff like. A game is said to be a constant sum game when the players' payoffs sum to a constant value. If these payoffs always sum to zero, the game is said to be a zero-sum game.[30] The zero-sum game can be expressed more precisely as follows:

$$u_1 (s_1, s_2)= -u (s_1,s_2),\ u_2 (s_1,s_2)= u (s_1,s_2)$$

This means that the game's dynamics lead to an outcome where one player's loss equals to the amount of other player's gain. As the players are assumed to be rational the losing player tries to minimize the value of the utility function $u(s_1,s_2)$ and the winning player tries to maximise it.[31]

In a two player game the winning player optimises his strategy $u (s_1,s_2)$ by selecting ;

$$\underline{V}(G)=\max_{s_2} u (s^*_1, s_2)< \max_{s_2} u (s_1, s_2),\ s_1 \varepsilon S_1$$

, where $\underline{V}$ is the game's outcome and the s^*_1 is a winning player's warranty strategy.

In a same manner the losing player optimises his strategy $u(s_1,s_2)$ by selecting ;

$$\overline{V} (G) = \min_{s_2} u (s_1, s^*_2)\geq \min_{s_2} u (s_1, s_2),\ s_2 \varepsilon S_2.$$

---

[29] Karelahti, op.cit

[30] Carmichael, op.cit. p. 12.

[31] Karelahti, op.cit.

If $\underline{V} = \overline{V}$, then;

$u\ (s_1, s_2) \leq u\ (s^*_1, s_2{}^*) \leq u\ (s_1, s^*_2)$. [32]

The above formulation describes the so called saddle point discussed earlier. Here each player achieves their maximum possible payoff by using their own warrant strategies (s^*_1 or s^*_2).

The introduction of Prisoners' Dilemma provides a starting point for understanding other, more complex games. Prisoners' Dilemma is a game theoretical classic that is being covered in the most game theory related publications. Prisoners' Dilemma is a static, two player non-zero sum game where the two players are both being accused of a crime.[33] If both prisoners confess, they will be sent to prison for five years. If neither of them confess they will be sent to prison only for one year due to a lack of evidence. But if one of them confesses he will be released without charges while the other will be imprisoned for ten years. The game has two players; P1 and P2 who share a common strategy space $S_1 = S_2 = \{D, C\}$, where D= deny, C= confess.

The players' utilities are therefore;

u_1 (D, D)= -1
u_1 (D,C)= -10
u_1 (C,D)= 0
u_1 (C, C)= -5

and

u_2 (D, D)= -1
u_2 (D,C)= 0
u_2 (C,D)= -10
u_2 (C, C)= -5

This normal form game can also be called a matrix game as the players' strategies can be presented in a row-column matrix. As both players' utilities can be placed into a same cell within the matrix, the matrix is commonly called a bi-matrix.[34] The following table is a Prisoners' Dilemma presented in a bi-matrix format.

---

[32] Karelahti, op.cit.
[33] Stearns, Maxwell L., 'A Beautiful Mend: A Game Theoretical Analysis of the Dormant Commerce Clause Doctrine', *William and Mary Law Review* 45.1, 19 May (2007) [online], Available from: http://www.questia.com/PM.qst?a=o&d=5002058582, [Accessed 19 April 2007].
[34] Gibbons, op.cit. p. 3.

		Prisoner 2	
		D	C
Prisoner 1	D	-1, -1	-10, 0
	C	0, -10	-5, -5

Based solely on the bi-matrix both players should select a strategy combination (D, D) as it would provide relatively short sentences for them both. What makes the strategy selection more difficult is the fact that the prisoners are not allowed to discuss with each other. And based on the principles of the game theory, both prisoners are only seeking to maximise their own benefits. Therefore, neither prisoner can trust on the other; both are unsure whether the other stays with the strategy D. Because of that uncertainty and the principle of maximising personal benefit, both prisoners must assume that the other is actually selecting strategy C. That is, by selecting strategy C, a prisoner would maximise his own benefit compared to that achievable with strategy D. As both prisoners follow the same thought pattern they equally end up selecting strategy C. This leads the players selecting a strategy combination (C,C) which is against their principle of maximising their achievable personal utility.[35]

The idea of Prisoners' Dilemma can also be described in a military context. For that purpose a game of two players is assumed with player Red (R) and player Blue (B). Both players are flying fast jets with similar performances. Players are engaged in a visual combat and are therefore involved in a game of perfect knowledge. During the fight, both players are losing both their potential- and kinetic energy and neither of the players have an offensive posture. Both players have three options; they can either increase or decrease their turn rate or to keep their turn rate constant. In order to simplify the game, the players decide either to increase or decrease the turn rate. If a player decides to increase the turn rate, it will provide the player with an almost immediate shot opportunity but on the other hand the manoeuvre will lead to a tremendous loss of overall energy. If the shot turns out to be unsuccessful the loss of energy will automatically lead to highly defensive position. If the player decides to decrease his turn rate he would start to gain energy (airspeed) that in turn could be later used to gain the offensive position. But doing so, the player is accepting a risk of being shot if the other player decides to increase his turn rate at the same moment. As a result of their manoeuvring and their aircraft performances both players are

---

[35] Harris, Andrew C., and Madden, Gregory J., Delay 'Discounting and Performance on the Prisoner's Dilemma Game', *The Psychological Record* 52.4 (2002) [online], Available from: http://www.questia.com/PM.qst?a=o&d=5000677784, [Accessed 19 May 2007].

constantly trading potential energy for kinetic energy i.e. they are descending. Both players need to avoid the ground and therefore they must at some point start to gain energy by reducing the turn rate. Hitting the ground would result in an inevitable loss of life and therefore both players favour the reduced turn rate. The game is assumed to be static, so both players make their decisions at the same moment. Players have two possible strategy choices. First, a strategy {P} (pull) to increase the turn rate, to attempt a kill and to accept the reduced energy level caused by pulling and second, a strategy {E} (ease) to reduce the turn rate, to accept risk of being shot at and to increase energy. The game can be expressed in a normal type game by writing players' utilities in a bi-matrix. In the following table the player B is situated in rows and the player R is situated in columns.

RED/BLUE	EASE	PULL
EASE	10,10	0,20
PULL	20,0	5,5

A player can achieve the maximum payoff by selecting 'pull', if the other player selects 'ease'. But if the blue player selects 'pull' also the red should select 'pull', as this increases the red's payoff from zero to five. If both players select 'ease' they both receive a payoff of ten. But if the red selects 'ease', the blue will select 'pull' as it increases his payoff from ten to twenty. As the red knows this, he will also select 'pull'. As a result, both players will select 'pull' even though they would both get higher payoffs by selecting 'ease'. It is said that the game has now reached the so called Nash equilibrium. The Nash equilibrium will be discussed later in more detail.

Next, the different principles how to solve games is clarified. When games are being solved it is vital to find and eliminate the dominated strategies. The following section will explain the principles of solving a game by eliminating the dominated strategies through iteration. In the Prisoners' Dilemma the basic assumption is that neither player is going to play the strongly dominated strategy as it would be irrational. Because of the strategy 'deny' is dominated by the strategy 'confess', neither player will select the former strategy. To further explain the elimination of dominated strategies another generic game is introduced. In this game there are two players, player P1 and player P2. The player P1 has a strategy space of S_1{up, down} and the player P2 has a strategy space of S_2{left, middle, right}. The bi-matrix below shows the players' imaginary utilities.

P1/P2	left	middle	right
Up	1,0	1,2	0,1
down	0,3	0,1	2,0

The bi-matrix shows that by selecting strategy {middle} the player P2 is in all cases able to achieve a higher utility than by selecting strategy {right}. Therefore it is said that the strategy {middle} dominates the strategy {right}. A rational player would never play the strategy {right} and it can be eliminated from the matrix as an irrational option. Following that assumption the bi-matrix can be redrawn as seen below.

P1/P2	left	middle
Up	1,0	1,2
down	0,3	0,1

From the edited bi-matrix it can be seen that the player P1's strategy {up} now dominates the strategy {down}. Following the previous logic, the P1's strategy {down} can now be eliminated as it is dominated. The resulting bi-matrix is seen below.

P1/P2	left	middle
Up	1,0	1,2

Once this is done it can be seen that the player P2's strategy {middle} dominates strategy {left}. Therefore, the strategy {left} can be eliminated from the P2's strategy space. The remaining bi-matrix is as follows;

P1/P2	middle
Up	1,2

The bi-matrix that is left includes the solution for the game (up, middle). Also, the bi-matrix shows the utilities (1,2) the players can achieve when playing the optimum strategy combinations. The process of eliminating the strategies through iteration is not always quite this straightforward. First, a player can be rational while making seemingly irrational strategy choices. That switches the game mechanics into complicated cheating principles. For simplicity, the game model formulated in this paper does not cover cheating principles. Still, some cheating principles are discussed later in general terms. Second, it is not enough that players believe that other players are rational. A player must believe that other players are rational and believe that the other players believe that the player is rational ad infinitum.[36] Third, in some cases the elimination of dominated strategies might give rather confusing picture of the game mechanics if there are no strongly dominated strategies to be found.[37] In order to enable the solving of games without strongly dominated strategies, a Nash equilibrium is introduced.

---

[36] Vega-Redondo, Fernando, *Economics and the Theory of Games* (Cambridge, England: Cambridge University Press, 2003) p. 31.

[37] Gibbons, op.cit. p. 7.

Nash equilibrium is solved when such a strategy can be found that it provides a player with a maximum utility despite the strategy choices other players might make.[38] In a n-player normal form game $G= \{S_1.....S_n; u_1.....u_n\}$ strategies $(s^*_1.....s^*_n)$ will form a Nash equilibrium for all players 'i' if;

$$u_i(s_1^*,\ldots ,s_{i-1}^*, s_i^*, s_{i+1}^*,\ldots ,s_n^*) \geq u_i(s_1^*,\ldots ,s_{i-1}^*, s_{i.}, s_{i+1}^*,\ldots ,s_n^*)$$

that is,

$$\max_{s_i \varepsilon S_i} u_i (s_1^*,\ldots ,s_{i-1}^*, s_{i.}, s_{i+1}^*,\ldots ,s_n^*)$$[39]

To clarify the idea of defining a Nash equilibrium, another imaginary game is introduced. The game has two players, player P1 and player P2. The player P1 has a strategy space $S_1\{T, M, B\}$ and the player P2 has a strategy space $S_2\{L, C, R\}$. For the purpose of understanding the Nash equilibrium, only the utilities resulting from different strategy combinations are important. The utilities and strategies are described in a following bi-matrix.

P1/P2	L	C	R
T	0,4	4,0	5,3
M	4,0	0,4	5,3
B	3,5	3,5	6,6

As it can be seen from the bi-matrix, there are no dominated strategies. Therefore, the game can not be solved by using the iterative elimination process described earlier. In order to solve this game, one must define the P1's best response to P2's strategy s_2;

$R1\ (s_2) =[\{s_1 I s_1\}=\text{argmax}\ s_1 \varepsilon\ S_i\ u_i\ (s_1,s_2)]$.[40]

The Nash equilibrium is reached when for each player there can be found a strategy that is the best response against other players' all possible strategies.[41] This means that;

$s^*_1= R_1\ (s^*_2)$ and $s^*_2 = R_2\ (s^*_1)$, and therefore

$s^*_1= R_1\ (R_2\ (s^*_1))$.[42]

---

[38] Carmichael, op.cit. p.36.
[39] Gibbons, op.cit. p. 8.
[40] Karelahti, op.cit.
[41] Linster, Bruce G. 'Evolutionary Stability in the Infinitely Repeated Prisoners' Dilemma Played by Two-State Moore Machines', *Southern Economic Journal* 58.4 (1992),[online], Available from: http://www.questia.com/PM.qst?a=o&d=5000143424, [Accessed 28 March 2007]
[42] ibid.

To make this more illustrative, the players' best responses against each other's strategies are underlined in the bi-matrix. As it can be seen from the following matrix, the Nash equilibrium can now be easily defined; a strategy combination that has both P1's and P2's strategies underlined {B,R} provides both players with the best response and, by definition, is the game's Nash equilibrium.

P1/P2	L	C	R
T	0,$\underline{4}$	$\underline{4}$,0	5,3
M	$\underline{4}$,0	0,$\underline{4}$	5,3
B	3,5	3,5	$\underline{6}$,$\underline{6}$

When the dominated strategies are eliminated through iteration, the process always leads to the Nash equilibrium. But on the other hand, if the Nash equilibrium is solved the same result may not be necessary achieved by using the elimination process.[43] Also, it must be emphasised that the Nash equilibrium is not necessarily the best option for all players. The games solved so far have all been static. This paper suggests that military operations are dominantly dynamic by nature. As the Nash equilibrium can not always be found in dynamic games, yet another game theoretical tool needs to be introduced.

The Stackelberg equilibrium is used when dynamic games are being solved. The idea of dynamic game assumes that players take turns to make decisions. Because of the sequential nature of the decision making the players in the dynamic games are either leaders or followers. In a dynamic game the players' position in and information about the game is not equal. This means that the leader always knows the utility functions of the followers. A dynamic game follows a pattern where the leader first takes the initiative i.e. declares his strategy choice. After that the followers act, or react to be more precise, based on leaders strategy.[44] At first sight it may seem that the leader would always gain unilateral benefit from his initiative stance. But as this paper will later point out this is not true in all cases.

When a player 1 is a leader the Stackelberg equilibrium (s^*_1,s^*_2) solves a problem;

$$\max_{s1 \varepsilon S1} u_1(s_1, R_2(s_2)).$$[45]

Based on the Stackelberg equilibrium's principle the leader selects a strategy that results in optimum utility when the followers try to maximise their own utilities.[46] The Nash equilibrium may not be found in a dynamic game. The Stackelberg equilibrium on the other hand can always be

---

[43] Gibbons, op.cit. p. 8.
[44] Karelahti, op.cit.
[45] ibid.
[46] ibid.

found as the utility is always unambiguous even if the strategies are not. [47] The basic assumption in military operations is that it is not harmful to have an initiative. In game theoretic terms it is never harmful to be a leader if the Nash equilibrium can be found. But if the Nash equilibrium can not be found it may be more beneficial to be a follower than a leader.[48] It is important to note that one might end up getting different results when solving either the Nash equilibrium or the Stackelberg equilibrium. This phenomenon can be explained with the fact that these two equilibriums assume different phasing of players' decisions and therefore suppose games of different dynamics.[49] For theoretical purposes in general and for practical military solutions especially it is worth emphasising the two logical outcomes of the equilibriums discussed. First, if the game has an equilibrium each player is willing to select a strategy dictated by the equilibrium. More clearly, each player is likely to select a strategy that is the optimum response if other players are using strategies suggested by the equilibrium. Second, a game with the described logic is self-reinforcing as no one is willing to deviate from their equilibrium based strategy. [50] To further explain the meaning of Nash and Stackelberg equilibriums a widely known game called Friends and Enemies is introduced.

The Friends and Enemies is a simple static game of two players that also rather well captures the nature of operational level military decision making. In the Friends and Enemies the players always seek to select opposite strategies as they have opposing preferences. For the purposes of this paper, it is assumed that military staff is evaluating two different friendly courses of action against two possible enemy courses of action. The game has two players; player Blue (B) and player Red (R). The player B is planning to launch an offensive air operation against a target. He can effectively use his assets to target either area (x) or area (y). The player R is a defender who can move his high value assets either to the area (x) or area (y). If the player B selects strategy {x} the player R always selects strategy {y}. If the player B selects strategy {y} the player R always wants to select strategy {x}. Based on their strategies the players can achieve utilities -1 or 1. The following bi-matrix shows players' utilities with different strategy combinations. Based on different strategy choices, the players' maximum utilities are underlined in the bi-matrix.

		Player R	
		x	y
Player B	x	$\underline{1}$, -1	-1, $\underline{1}$
	y	-1, $\underline{1}$	$\underline{1}$, -1

---

[47] Virtanen, Kai, *Optimal Pilot Decisions and Flight Trajectories in Air Combat*, (Helsinki: University of Technology Systems Analysis Laboratory), Research Reports A90, (2005).
[48] ibid.
[49] ibid.
[50] Poropudas, op.cit. p. 28.

As can be seen from the bi-matrix, there are neither dominating strategies nor the Nash equilibrium to be found. Therefore, if players are using pure strategies it is basically impossible to predict how the game would unfold. But as military planning and decision making puts great emphasis on initiative, the Friends and Enemies is not necessary a static game. In order to capture the nature of the military reality more clearly it is assumed that the game is dynamic i.e. players are making decisions sequentially and that the player B has the initiative. This leads to dynamics where the player B is a leader and selects either strategy {x} or {y}. After that the player R as a follower makes his own strategy decision. As the follower always wants to select a different strategy than the leader he has the advantage despite the fact that the leader has the initiative. It seems clear that in this case being a leader is harmful, especially if the players know each other's decisions. This notion is not necessarily in conflict with the fact that the initiative is widely accepted as desirable in a military context. In order to gain an advantage while taking the initiative at the same time the leader must use cheating techniques, or deception in military terms. This means that the leader should try to mislead the follower about his decision. Even when using the cheating principles the leader should randomise his strategies to some extent as otherwise cheating would lose its effectiveness. Once the players start to randomise their strategies the game can be solved. All normal form games, including zero-sum games, have a Nash equilibrium if the players are randomising their strategies i.e. using mixed strategies.[51] The next step when formulating the military decision making problem as a game, is to introduce the concept of game tree.

A game tree is a graphical illustration of the game. It starts from the leader's first decision point also known as the root node. The different decision options create branches emerging from the decision node. The followers then make their decisions at their respective decision nodes thus creating branches that equal the number of their decision options. In each decision node one of the players is active i.e. he has a turn to make his decision. Once the last decision is made, the game proceeds to a final node also known as a leaf node. From this leaf node it is possible to define the players' overall utilities.[52] For the purposes of this paper it is important to note that a node can also include probabilities that affect the subsequent outcomes.[53] Such a node is commonly called a nature node and normally requires some level of simulation to be implemented effectively. The effects of the nature node are discussed later in more detail as the nature nodes greatly affect the effectiveness of the game theory's military applications. In general terms the game tree defines;

a) Players involved in a game.
b) Timing of decisions i.e. turns that create the dynamics of the game.
c) Players' decision options i.e. their strategy spaces.

---

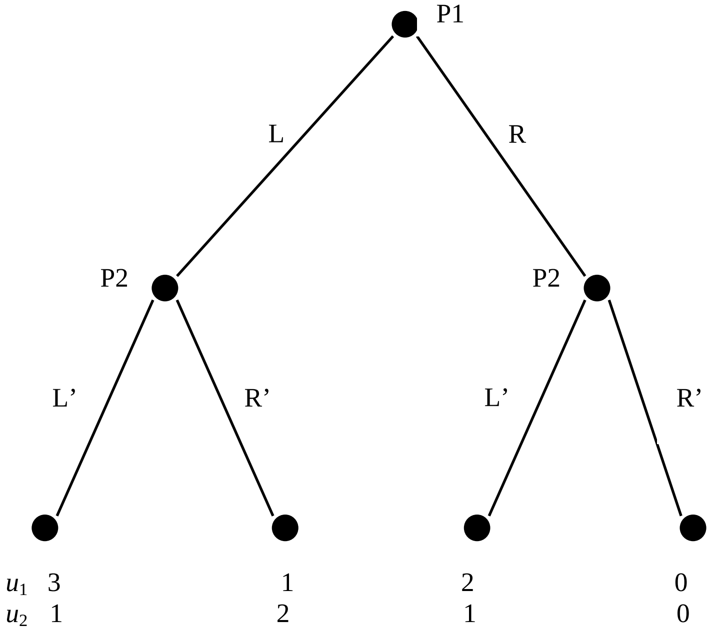
[51] Carmichael, op.cit. p. 49.

[52] Rullière, Jean-Louis and Walliser, Bernard ed., *7 From Specularity to Temporality in Game Theory, Game Theory and Economic Analysis*, (London: Routledge, 2002) pp.137-140,[online], Available from: http://www.questia.com/PM.qst?a=o&d=103310867, [Accessed 30. March 2007].

[53] Poropudas, op.cit. p.32.

d) Information a player has while making his decision.
e) Players' utilities that follow from decisions.[54]

To clarify the idea of the game tree, an example of the game described as a game tree follows. In this game there are two players, player P1 and player P2, both with perfect information and perfect memory. The tree starts with a root node where the P1 selects his strategy from the strategy space of {L, R}. At this stage the strategy selection does not take into account the player's utilities. Instead, it only defines the unambiguous actions of the player.[55] The P2 sees the decision made by the P1 and then selects his strategy from the strategy space of {L', R'}. After the P2 has made his decision the game ends in leaf nodes. The different leaf nodes show all the possible strategy combinations. Also, from the leaf nodes it can be seen how the different strategy combinations affect players' utilities; $u_1(s_1, s_2)$ and $u_2(s_1, s_2)$. The game tree with different strategies and utilities is shown below.



In this game the P1 has strategies (L) and (R) available but the strategies the P2 has available are;

a) L' if P1 plays L and L' if P1 plays R = (L',L).
b) L' if P1 plays L and R' if P1 plays R = (L', R').
c) R' if P1 plays L and L' if P1 plays R = (R', L').
d) R' if P1 plays L and R' if P1 plays R = (R',R').

---

[54] Gibbons, op.cit., p. 116.
[55] Virtanen, op.cit.

Even though this game is extremely simple with minimum number of choices, it shows the complexity of different strategy combinations. When military planners are evaluating different courses of action, the number of possible strategies is seldom this limited. As the number of the decision options and the game's levels (player's decision turns) of the game increase, the game tree starts to expand almost exponentially. For the purposes of this paper the previous example of the dynamic game is next described as a normal form game in a bi-matrix.

P1/P2	(L',L')	(L',R')	(R',L')	(R',R')
L	3, 1	3,1	1,2	1,2
R	2,1	0,0	2,1	0,0

As the above bi-matrix suggests, the bi-matrix format is not a very illustrative way of describing dynamic games. Therefore a dynamic game is typically described in a form of a game tree. This is mostly a practical issue as each game can be described either as a bi-matrix or as a game tree. But if the game becomes very complex the bi-matrixes can be used to illustrate limited parts of the game, such as a separate decision or nature nodes.

When a military operation is described as a game, the players seldom have complete information or a perfect memory. In game theory, a concept of information group is used to describe players' incomplete information in dynamic games and to capture the static nature of the game when it is presented as a game tree. Next the information groups will be discussed in some more depth.

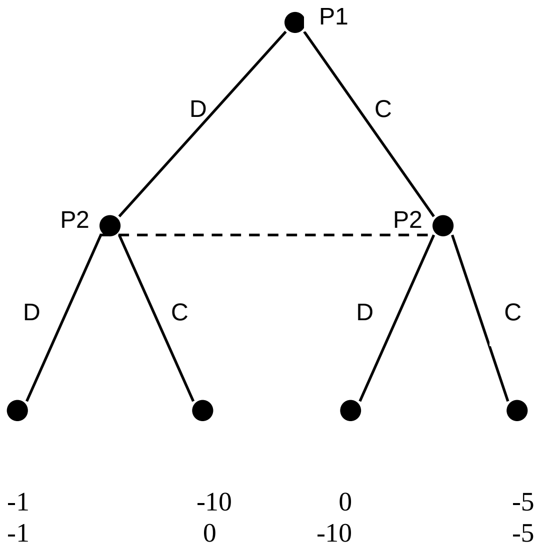
The information group can be defined as a group of decision nodes where;

a) A player has a decision to make in each of the nodes.
b) A player does not know the specific node within the information group he is at. [56]

When a game tree is created, the information group is described with a dotted line or ellipse connecting the nodes belonging to that information group. The picture below shows again the game of Prisoners' Dilemma – this time in a form of a game tree. As the game tree suggests, the player 2 does not know the decision node he is at.

---

[56] Karelahti, op.cit.



When both players know the node they are at there are no information groups and no inter-node lines are drawn. Such a game assumes that all players have a complete memory. To be exact, even a single decision node can form an information group. This specific case of information groups is called a ‘singleton information set’.[57]

To enable more complex game trees to be drawn and solved, concepts of sub-game and sub-game Nash equilibrium are introduced. In a dynamic game a sub-game begins from the decision node that is a singleton information set. A sub-game covers all the decision- and leaf nodes originating from that node.[58] The root node, however, can not be the beginning node of a sub-game. Also, a sub-game may not cross different information groups. Closely related to the sub-games is a Nash sub-game perfect equilibrium.[59] The Nash equilibrium is said to be sub-game perfect when player’s every strategy within the sub-game forms a Nash equilibrium.[60] The reason the Nash sub-game perfect equilibrium is essential both to this paper and to military decision making in general is that it provides a tool to eliminate player’s strategies that on one hand seem harmful but on the other hand are not credible. [61]

In the examples discussed so far, players’ utilities had unambiguous values as they reached the leaf nodes. Unfortunately, sometimes utilities can not be measured in such a straightforward manner as different players may have different preferences towards otherwise equal outcomes. The different preferences can be taken into account by weighting the pure outcomes. The

---

[57] Gibbons op.cit. p.121.

[58] Gale, Douglas, *Strategic Foundations of General Equilibrium: Dynamic Matching and Bargaining Games*, (Cambridge, England: Cambridge University Press, 2000) p. 56.

[59] Edward M. Graham, Market Structure and the Multinational Enterprise: ’A Game-Theoretic Approach’, *Journal of International Business Studies* 29.1 (1998), [online], Available from: http://www.questia.com/PM.qst?a=o&d=5001338111, [Accessed 19 May 2007].

[60] Karelahti, op.cit.

[61] Virtanen, op.cit.

weighting is done by using utility functions that link together both the player’s pure outcome and his preferences towards it. The formulation of utility functions is probably the most crucial point when applying game theory to military decision making. If the players’ preferences are measured incorrectly, the game will provide misleading information about the game’s results. The utility functions shall be discussed in more detail as the actual game is formulated.

The next element of the game formulation to be clarified is the concept of probabilities or chance nodes. In a dynamic game the chance nodes are used to express the stochastic nature of the game. Due to the probabilistic nature of such games, there are multiple possible outcomes for a single decision. And at every level of the game those multiple outcomes generate their individual branches for the game tree as the game evolves. When probabilities are added into a game it soon becomes very complex as the probabilities generate ‘parallel realities’ that are all true except that they exist with different probabilities. But the stochastic nature of the game is as much relevant as it is complex. As long as the weapons can not be delivered with 1.00 probability of effect the probabilities can not be ignored. As military power is currently used also for non-kinetic effects, the measuring of probabilities becomes even more complicated.

As the rational players are trying to maximise their personal utilities, they have to take into consideration the uncertainties related to achieving those utilities. At its simplest, the players calculate the probabilities and utilities included in the game and then select the strategies that provide them with the maximum expected utility. But just as the players may have different preferences towards the game’s outcomes, they may also have a different attitude towards the risks caused by probabilities. The game theory differentiates three main types of players based on their risk behaviour. First, there is a risk loving player who seeks high gain despite the low probability it may have. Second, there is a risk averse player who selects strategies that provide lower gain but do that with a higher probability. Such a player generally seeks to minimise his losses even if it happens at a cost of lost benefits. Third, there is a risk neutral player whose risk behaviour falls somewhere between the previous two.[62] The risk behaviour strongly affects the behaviour of the player. Unless the player’s risk attitude is correctly understood and implemented into game dynamics, the player’s rationality shall not be captured correctly. For a military planner this may bring unpleasant surprises if the reality does not mirror the assumptions made during the planning phase. It should be fair to argue that one of the biggest challenges in game theory is the correct formulation of the player’s risk behaviour. But it has to be said that for the military commander the challenges remain the same whether the game theory is used to enhance the intuition or not.

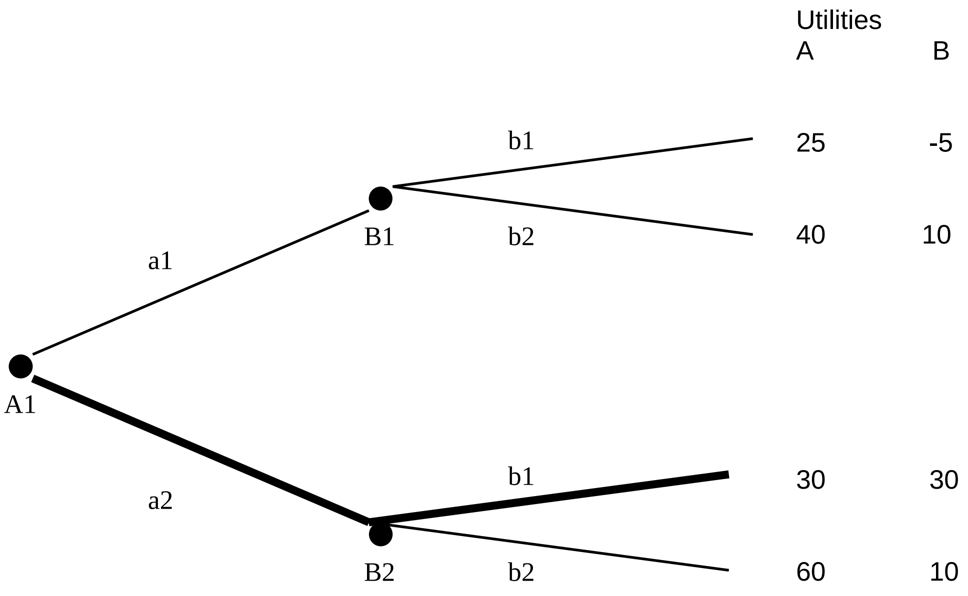
---

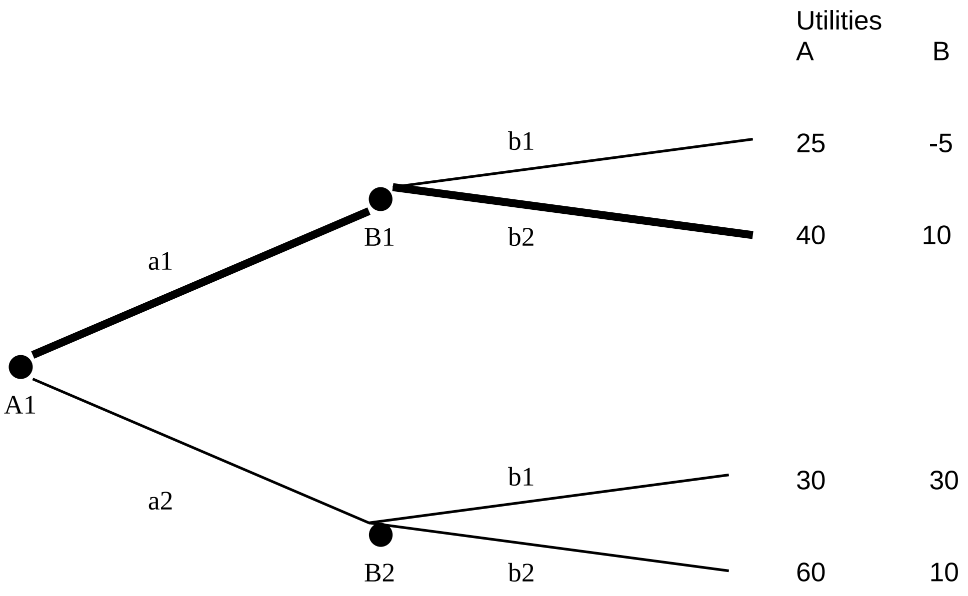
[62] Coram. op.cit., p. 14.

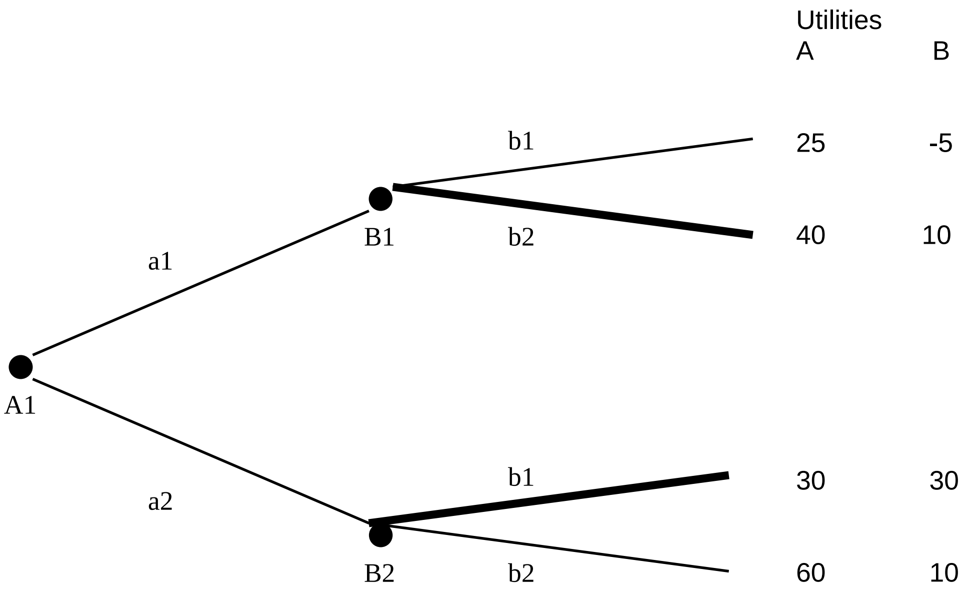
The utility functions and game trees are not an end themselves despite the fact that those may help the military planner to conceptualize the problem at hand. But the game theory looks beyond that obvious benefit and uses them to find players' optimal strategy combinations. This is done by using a backward induction. The backward induction is used to solve games where players make selections between several Nash equilibriums. The idea here is to go through the game tree starting from the leaf nodes, to check each sub-game's payoffs individually and to proceed systematically towards the root node. It is then possible to define the strategy combinations that provide players with the best outputs. What is essential for the backward induction is its ability to relieve harmful strategy choices. If a strategy choice is unilaterally harmful it can be used as a coercive or persuasive tool.[63] But if the strategy choice is mutually harmful neither the coercion nor the promise to play that strategy is credible. This is because a player will not select a mutually harmful strategy if he is able to increase his personal utility by selecting another strategy.[64] A simple two-step game is introduced to clarify how the backward induction is used. A game has two players, a player A and a player B. The players have complete memories and players' utilities are assumed to be common knowledge. At a decision node A1 the player A has a strategy space of {a1, a2}. The decision made by player A is seen by player B. Following the A's decision the player B makes his decision in nodes B1 and B2 respectively. The following pictures illustrate the game with different strategy choices. In the pictured game trees the player A's decision is first bolded followed by the bolding of the player B's optimal response to A's strategy. The first game tree shows player B's best response when A selects {a2} and the second game tree shows player B's best response when A selects {a1}. The third picture shows player B's optimal responses against any strategy choice made by player A.

---

[63] Kreps, David, M., *Game Theory and Economic Modelling*, (Oxford: Clarendon Press, 1990), p. 50.
[64] Carmichael, op.cit. p.86.


Utilities
A
B
b1
25
-5
B1
b2
40
10
a1
A1
b1
30
30
a2
B2
b2
60
10

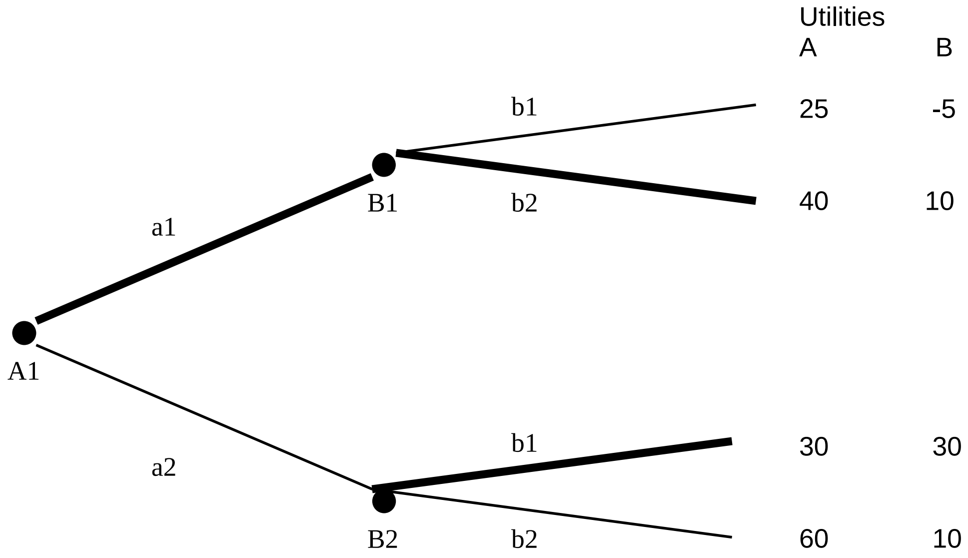

Utilities
A
B
b1
25
-5
B1
b2
40
10
a1
A1
b1
30
30
a2
B2
b2
60
10



The player A can and will go through the game options seen above even before he makes his initial decision. The player A will see that the sub-game B2 has no Nash sub-game equilibrium. This means that player B's promise to play strategy {a2,b2} is not credible. Once player A has done that observation the game becomes greatly simplified. From the player A's perspective, the situation is getting closer to a decision analysis. Player B is now completely reactive and player A's only task is to find the best decision. Based on the game's dynamics the player A presumably recognizes that the utility '60' is out of his reach. Instead, he sees the utility '40' as achievable because the sub-game B1 has a Nash sub-game equilibrium.[65] The following picture shows the player A's best strategy against the player B's alternative optimal strategies {a1, b2} and {a2, b1}. The picture illustrates the game's solution using the backward induction. The solution includes both the players' utilities and the strategy combinations used.

---

[65] Carmichael, op.cit. pp. 87-89.



Next the previously described military decision making problem shall be formulated as a game. The imaginary Joint Task Force Commander's question was; *'If the enemy is able to allocate four air defence fighters to the area 'XX', what is the expected outcome against target 'YY' when four friendly air superiority fighters and an air interdiction unit is used?*'' When game theoretic terms and logic is used the problem sounds somewhat different. The game theoretic version of the given problem would be: *'How does the risk behaviour and strategy choices influence the outcome of the military operation when it is described as a two-player game with both players seeking to optimise their strategies'*. For the game theoretic analysis to be effective relatively precise and detailed information about the problem is required. For simplicity the following assumptions are made;

a) Both players can select between three different tactics. Each tactic includes a different risk and a different effectiveness of weapon employment. The risk and the weapon effectiveness are linked so that higher the risk, higher the effectiveness of the weapon employment and vice versa.

b) The game consists of series of engagements between players i.e. the game is first split in short parts which are then drawn together in the end. Players are free to choose their tactic before each engagement. Players stay within the selected tactic throughout the engagement. By iterating all the sub-games (engagements) with all different tactics combinations it is possible to define what strategy combination optimises players' payoff.

c) In order to illustrate different strategic level risk behaviours both players are given a value for the mission level risk acceptance. Each game is ran using just a single risk behaviour. By iterating the games (series of engagements) with different higher level risk behaviours it is possible to define what risk attitude optimises players' payoff. The higher level risk behaviour and the players' tactic selection logic are discussed in grater detail later in the paper.

Based on the above assumptions and commander's task the actual game can be formulated. The game has two opposite sides that both have five players. The opposing sides are given names blue (B) and red (R). Each individual player is called a 'pod'. The players can be identified as each pod has been given a player specific number. Blue uses pods from #1 to #5 and red uses pods from #6 to #10. The red pod #10 is a static ground based player against whom the blue players are trying to deliver a kinetic effect. Red pods #6…#9 are air defence fighters trying to protect pod #10. The blue pod #5 is simulating a larger air-to-ground attack unit that is seeking to achieve a kinetic effect against pod #10. Blue pods #1…#4 are the air-to-air fighter sweep unit tasked to protect the pod#5.

The players have different preferences towards risk. In this paper, the players' differences are described using the concepts of risk behaviour and players' tactic selection. The risk behaviour is a performance model given to the whole operation by higher authorities and the tactic selection logic presents the players' available strategy space. The higher level risk behaviour affects the players' available strategy space as shall be shown later. The risk behaviour (W) is defined as follows;

Wkb+Wlb=0,1,

where different risk attitudes produce the following weights;

LO RISK (risk averse)= 0.01 (kb) + 0.09 (lb)
MEDIUM RISK (risk neutral)= 0.05 (kb) + 0.05 (lb)
HIGH RISK (risk loving)= 0.09 (kb) + 0.01 (lb), where

(W) = the weight describing the risk behaviour
(kb)= kill blue (air to air kill achieved by blue pod)
(lb) = loss blue (air to air loss suffered by blue pod)

As can be seen from above, a risk averse player shows little interest towards achieved kills and is more concerned about his own losses. A risk neutral player gives equal weight on kills and losses, whereas a risk loving player has opposite preferences than the risk averse player. The same principle is used to describe the risk behaviour of the red pods. When doing so the ‘kb’ is replaced by ‘kr’ and the ‘lb’ is replaced by ‘lr’. It is also necessary to take account the individual players’ preferences towards the destruction of the protected target or the kill of the air interdiction unit. Both players are given a weight W# that describes the mission success. W# is 0.9 for both players. Therefore the sum of all weights together is; Wbk(r)+Wbl(r)+W#=1. But wins and losses are not enough to describe players’ overall preferences. Players may have very different subjective preferences about what a kill or a loss is worth of. For that purpose, both players have two single-attribute utility functions to describe these preferences;

For blue: 0 bk ->Ubk 0, 4 bk->Ubk 1 and 0 bl->Ubl1, 4 bl->Ubl 0

For red: 0 rk->Urk 0, 4 rk->Urk 1 and 0 rl->Url1, 4 rl->Url 0

Therefore, if an engagement results in two red losses and one blue loss the utility functions would be:

- Ubk 0,5 and Ubl 0,75
- Urk 0,25 and Url 0,5

From the above utility functions it is now possible to define the overall utilities in the following manner;

For blue: Ub (bk, bl) =Wbk*Ubk(bk)+ Wbl*Ubl(bl)+ W#*(pod#10), and

For red: Ur (rk, rl) =Wrk*Urk(rk)+ Wrl*Url(rl)+ W#*(pod#5), where

k=kill
l= loss
b= blue
r= red
U= utility
bk= blue kill= blue victory
rk= red kill= red victory
bl= loss blue
rl= loss red
W=risk behaviour
Wbk+Wbl+W#=1

Wrk+Wrl+W#=1

The W# always has a value of 0.9 and it is multiplied either by 0 or 1 based on whether the player is able to gain success against his main objective; pod #10 for the blue player and pod #5 for the red player. [66]

For clarification an example follows. The blue player kills all four red players and eventually destroys the pod#10. Blue player is risk neutral and red player is risk loving. Therefore the utility functions would be;

For blue: Ub (4,0) =0,05*1(4)+ 0,05*0(0)+ 0,9*(1), and

For red: Ur (0,4) =0,09*0(0)+ 0,01*0(4)+ 0,9*(0)

The introduction of #W necessitates several rules to make the game workable. Based on common sense both players are given utility 0,0 (W#*0,0) in a case where the sub-game's outcome in respect of W# is ambiguous. Such an event would take place when players still have fighters left, pod#5 is not yet killed and pod#10 is not yet destroyed. If a sub-game's leaf node results in the loss of all of a player's fighters, the other player will get utility W# *1,0= 0,9. If after the leaf node all fighters from both sides are claimed dead, the blue player will get a utility W# *1,0= 0,9 assuming that pod#5 is still unharmed. Using these formulas and assumptions the game tree can be built in a rather straightforward manner by using players' losses and victories as utilities. As the game tree is finished, players are given a risk attitude. This risk attitude also defines the utility functions for both players.[67] Each chance node's outcome is put into a utility function as shown earlier. Once this is done it is possible to calculate the weighted overall utilities for both players.

The chance nodes' results will only be valid with some probability and those probabilities produce provisional realities. The overall utilities of provisional realities must be taken into consideration even when the probability of their existence is small. The following example illustrates how the overall expected utility (EU) is defined for the blue player. In this example the blue player uses strategy combination SS and the red player uses strategy combination ss; {SS,ss}. It is assumed that the sub-game's chance node results to kill-loss outcomes of; 2-1 with the probability of 0,7, 1-0 with the probability of 0,2 and 0-1 with the probability of 0,1. The expected overall utility that takes account all the probabilities can then be described as;

---

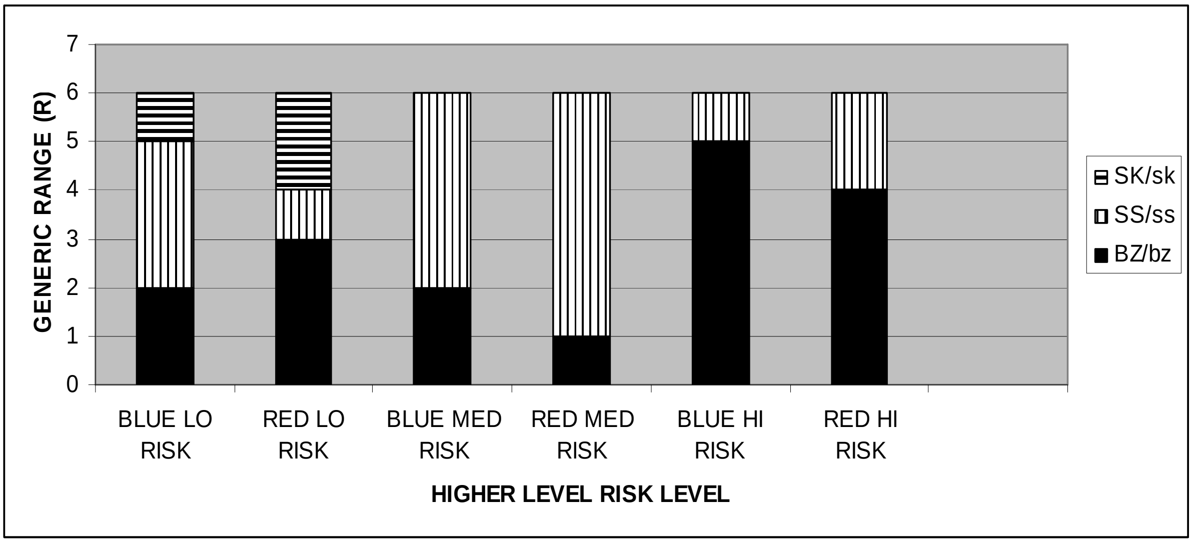
[66] Douglas Gale, *Strategic Foundations of General Equilibrium: Dynamic Matching and Bargaining Games* (Cambridge, England: Cambridge University Press, 2000) p.145, [online], Available from: http://www.questia.com/PM.qst?a=o&d=105724281, [Accessed 15 April 2007].

[67] Karelahti, Janne, Kai Virtanen, and Tuomas Raivio, *Game Optimal Support Time of a Medium Range Air-to-Air Missile*, (Helsinki: University of Technology Systems Analysis Laboratory), Research Reports, 2004, p. 2.

$$EUb\{SS,ss\}=0.7*Ub(bk,bl)+0.2*Ub(bk,bl)+0.1*Ub(bk,bl)+0,9*Ub(pod\#5)$$

Using the same logic it is possible to calculate the expected utility for the red player, EUr. It should be noted that this paper's formulation assumes that initially only the victories and losses are taken into account. Once this is done the victories and losses are weighted using the utility functions. The provisional realities resulting from chance nodes are taken into account by calculating their expected overall utilities. It is this expected overall utility that the players are trying to maximise.

What is still to be defined is the logic of how an individual player or pod selects between different tactics. Different tactics include different levels of risk and different expectations about the exchange ratio. The accepted level of risk and therefore the available tactics shall be defined based on a player's position in the play area. The closer the blue gets to the red pod #10, the higher risk the red player is willing to accept and vice versa. Respectively, the closer the red pods get to the pod #5 the higher risk the blue player is forced to take. The following picture describes tactics available for players. The blue player has tactics SK, SS and BZ available to him. The red player on the other hand has tactics sk, ss and bz. The tactics SK/ sk have the lowest risk, tactics SS/ss have a medium risk and tactics BZ/bz have the highest risk. Players risk behaviour is affected in two ways. First, players define their risks and available tactics based on the players' relative position in the game area. The left hand column in the picture below shows the generic range scale affecting the risk behaviour. With longer ranges, players are always select less risky tactics. As ranges decrease players are forced to select more efficient and risky tactics. Second, players are given higher level risk attitudes as discussed earlier. This risk attitude is seen in the picture's bottom row as (BLUE/RED) LO RISK, (BLUE/RED) MED RISK and (BLUE/RED) HI RISK. This higher level risk attitude is a reflection of higher level preferences about losses, wins and mission's importance. For the individual player this risk attitude dictates the available strategy space. For example, LO RISK blue player at range R 5,5 can use low risk tactic SK. If the same player would be given a MED RISK he would have to use more risky tactic, SS at the same range. But if the range would be 0,9 each players would have to choose the most riskiest tactic no matter what the higher level risk behaviour model is.



In the above picture the black bars show the players' high risk tactic (BZ,bz), the vertically striped bars show the medium risk tactic (SS,ss) and the horizontally striped bars show the low risk tactic (SK,sk). For example, once at R5 both players can use low risk tactics if the higher level risk level is LO RISK. But if the higher level risk level is MED both players will have to use tactics (SS,ss) at R5. Each player may be operating with different risk levels. Therefore, if blue player has LO RISK and red player has HI RISK they would have to use tactics (SS,bz) when at R3. Also, each individual pod makes their tactic selections individually within the boundaries of the given higher level risk levels. For example, pod#1 may be forced to use BZ while other blue pods may still have enough range available to select a tactic SK. Players try to select as low risk tactic as available range and higher level risk attitude permits. If the range available is not enough to execute a tactic dictated by the higher level risk attitude, the pod will disengage. Once the players' relative positions allow the use of higher risk tactics (requiring shorter range) the players will re-engage.

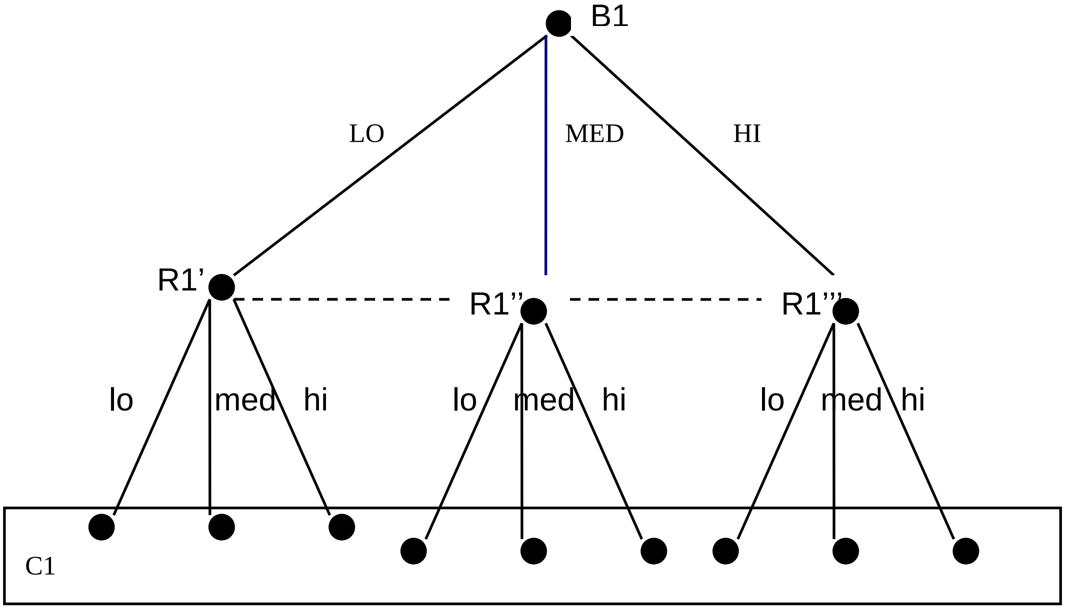
As the essential rules of the game have now been discussed, the attention is next drawn to the dynamics of the game. Game theoretic models can be categorised by different ways. In zero-sum games the utility achieved by a player automatically reduces to that of the other players. The other way of categorising games is to look at the game's dynamics. A static game can be considered consisting of just one acting or moving phase. A dynamic game on the other hand consists of several moves which may happen in phases. Also, the order at which the players make their decisions have an impact on the game's outcome.[68] The game formulated in this paper is dynamic. In a dynamic game it is essential that although players can make their moves simultaneously, the decisions dictating the moves are not made simultaneously. As discussed earlier players have either a 'leader' or a 'follower' status. The leader – follower position is not static. As the tactical situation constantly changes during the game, so can the leader and follower statuses be changed. The game in this paper assumes players as having complete memory and perfect information;

---

[68] Poropudas, op.cit. p. 15.

players know each others preceding strategy selections and the resulting utilities. Also, players know in which branch and node of the game tree they are currently at.

A separate stage of the game comprising of both players' decisions and subsequent chance node is called a 'level'. Levels are numbered in ascending order. The game begins with a decision made by the blue player at level one. Both players have four active pods (fighters) and each pod independently selects its strategy from the available strategy space. As a reminder, the available strategy space is influenced both by pods positions in the game area and the risk level given for the operation. The red player reacts to the blue player's strategy choice once all the blue pods have done their individual strategy selections. The red pods select their strategies using the same logic as the blue. Once the red has done his decisions the strategies for a single engagement have been decided. Based on this information it is possible to draw the level 1 of the game tree as seen below.

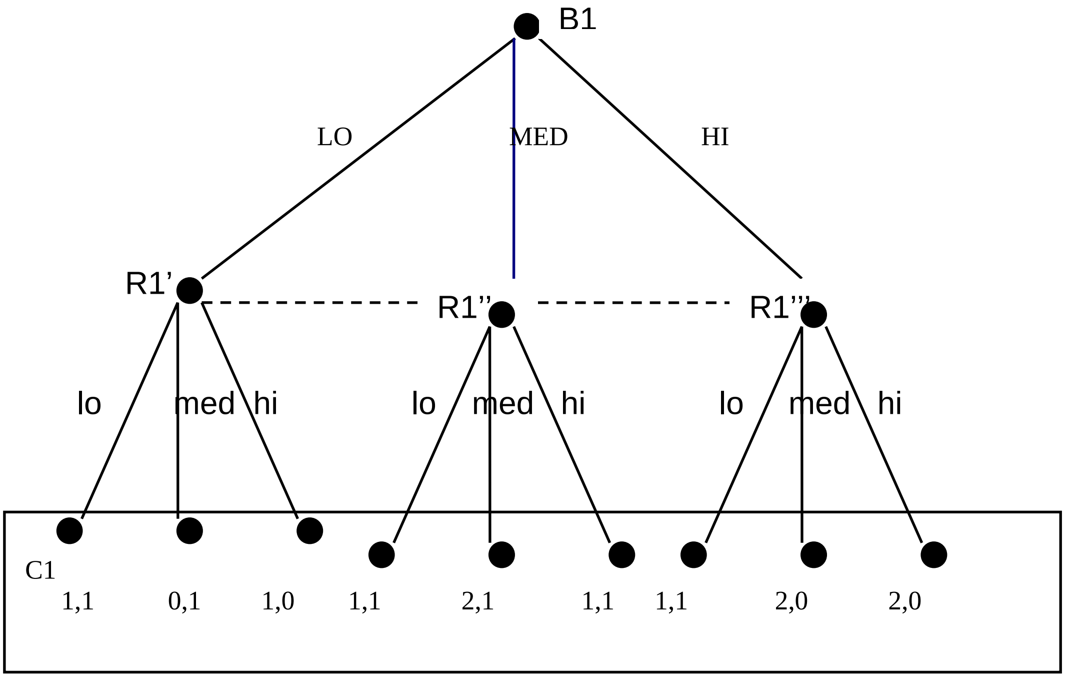


The game tree clearly shows how the blue player (B) has three optional strategies {LO, MED, HI} at his first decision node (B1). Once the blue player has made his decision, the red player (R) will know the node he is at. At this stage there are three possible nodes for red; R', R'' and R'''. In each of those nodes the red player can make three different strategy choices {lo, med, hi}. Once both players have done their strategy choices the game proceeds to the first chance node (C1). By using simulation with 100 monte carlo[69] runs (iterations) it is possible to effectively and comprehensively test all possible strategy combinations; {LOlo, LOmed, LOhi, MEDlo, MEDmed, MEDhi, HIlo, HImed, HIhi}. As an outcome the chance node produces engagement's wins and

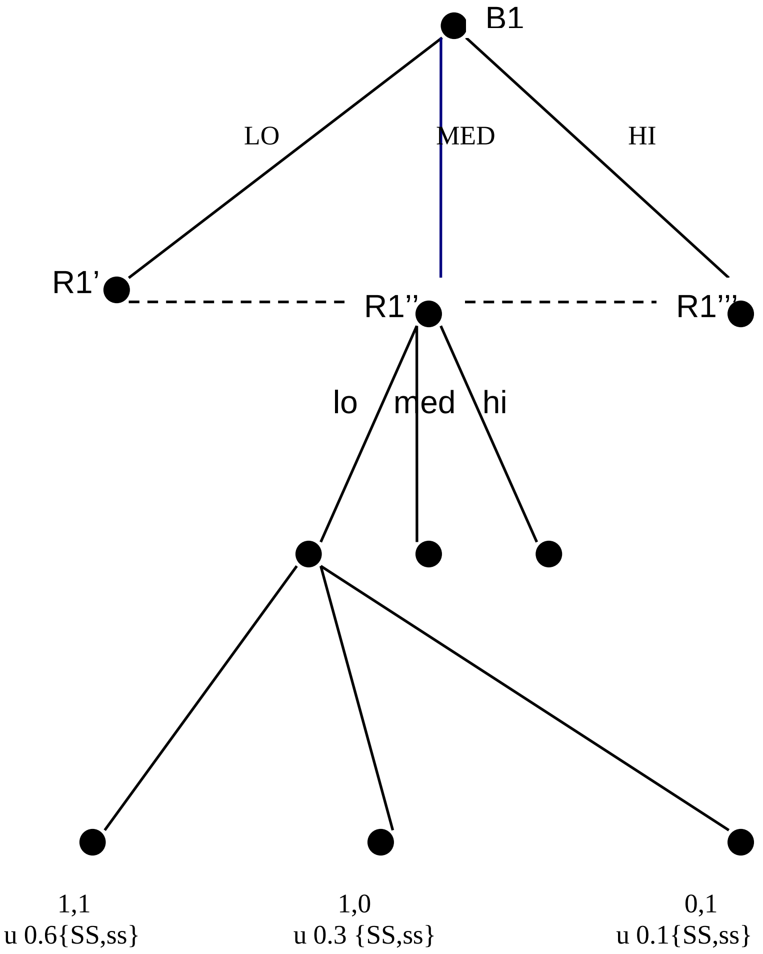
---

[69] No name, *ACEM operator's guide, version 4.5F*, (Raytheon Missile Systems, Operations Research Department, 2003).

losses for both players. As these outcomes are inserted into the game tree it would look as seen in the picture below;



The level one of the game tree has produced nine different outcomes. From here the sub-game specific utilities of each outcome can be defined by using the utility functions described earlier. The chance nodes' outcomes are true with certain probability. By using the probabilities of the simulation outputs it is possible to define the players' expected utilities for each sub-game. Just to illustrate the complexity of this game it is assumed that the outcomes from the nine different chance nodes all have three possible outcomes that occur with probabilities of 0.6, 0.3 and 0.1 respectively. Therefore, for each leaf node at level one, there are three provisional realities for the game to evolve to the next level. It can be seen that after the first decisions there are already 27 provisional realities. And all these realities produce further branches as the game evolves. The picture below illustrates the effects of probability generated alternative realities based on just one strategy combination {SS,ss}.



As basically all the chance nodes include some level of uncertainty, it is impossible to build the complete game tree in advance. Instead, the game tree has to be built level by level so that branches following each chance node are being taken into account. By using the probable outcomes of each chance node, it is possible to use backward induction for identifying players’ expected overall utilities. All possible chance node outcomes are accepted as starting nodes for the next level. The branches created this way present their respective realities with some probability. As the game evolves to subsequent levels, these probabilities are being cumulated. When the game’s overall outcome is defined, all the probabilities included in different sub-games are taken into account. It is for the game theorist to decide whether the outcomes with extremely low probabilities are accepted as the next level’s starting node or not. Once the game has evolved to the next level the process described here is iterated. The iteration is continued until the game ends branch by branch. The game described in this paper can end in two different ways, pod #5 destroys pod #10 and exits the play area unharmed or red pods destroy pod #5. In the latter case the blue player’s mission is considered as unsuccessful.

For the game's complete solution the cumulative effects of sub-games' monte carlo runs have to be studied. Using those findings it is possible to collect observations about the realisation of larger entity. [70] [71]

Once the game tree is complete, the game can be solved using backward induction. The solving begins from each leaf node and proceeds towards the root node. From the complete game tree it is possible to search for strategies that satisfy Stackelberg equilibrium. These strategies, by definition, are those that provide player with maximum payoff as follows;

(s^*_1 , s^*_2):

$$\max_{s_1 \varepsilon S_1} u_1(s_1, R_2(s_2))$$

The pure kill – loss calculations are no longer used during the backward induction. Instead, calculations are from here on based on expected utilities. As a solution for the game, and for the commander's military problem, the players' optimum risk level, optimum strategy combinations and expected utilities can be defined. Due to the probabilities involved, the game's solution is not absolute. Therefore the chance nodes' probabilities must be included in calculations of expected utilities. If the previous game tree and probabilities involved in it is used as an example, the solution of chance node's expected utility for the blue player would be;

$$EuB\ \{SS,ss\}= 0.6^* Ub(1,1) + 0.3^* Ub(1,0) + 0.1^* Ub(0,1)$$

The logic is same for the red player. Once the expected utilities are defined for each chance node in the game tree, it is possible to define the players' best possible strategy in each phase of the game. Once the best possible overall strategy is defined, the game is solved.

The following describes a solution for the game formulated earlier. First of all, the solution is an example that serves as a demonstrator only. Second, the solution is only partial as the complete solution would be too extensive for the purposes of this paper

A simulation program is normally used to solve the output of the game's chance nodes (C). In this paper the chance nodes were solved using a Unix-based Air Combat Evaluation Model (ACEM) program. The simulation could have been done using any other program suitable for the purpose. The actual simulation program inputs are not covered in this paper as they are considered irrelevant and too system specific for the paper's purposes. But just to get and idea of the

---

[70] Poropudas, op.cit., p. 13.

[71] Virtanen, K., Raivio, T., and Hämäläinen, R. P., 'Decision theoretical approach to pilot simulation', *Journal of Aircraft* 36, number 4,paper 0021-8669, Universal Technology Corporation, 1999, p. 632-641.

complexity of the simulation process it is worth mentioning that the creation of reliable simulation is often more challenging than the theoretical formulation of the game. The more realistic the simulation program is the more detailed and numerous the required inputs are. Not to mention all the background calculations related to missile fly out models, radar cross section models and aerodynamic models required to make the simulation tool even work.

The game was simulated from the root node to each subsequent leaf node. Due to the limitations of the ACEM program, the scenario had to be run in short steps. The actual steps are not shown here as they do not provide any additional value for this paper. The chance node's outcomes were recorded in the form of a game tree. The sample solution was taken down to the third level of simulations which also resulted in the first leaf node. Each simulation was tested with100 monte carlo runs. From the outputs of those runs the outcomes of the chance nodes were defined. In most cases the outcomes ranged across numerous probabilities. For simplicity, only the three most probable results were recorded. As a result, the missing information made the comprehensive use of the backward induction impossible. But as only a small part of the game tree was solved anyway, this was not considered to be a problem. If the game tree would be completely solved, all the possible strategy spaces would need to be included in calculations. Let us assume that in that case the chance nodes' outcomes would range across three different probabilities. Then the game tree would have 243 branches just after the second chance node (assuming that none of the chance nodes had turned out to be a leaf node). The number of branches would further increase if the probabilities would vary more and if the game was solved using multiple risk levels.

For the purposes of this partial solution example, only the strategy combinations resulting from the higher level risk attitude of {LO,hi} were solved in every decision node. Also, only the most probable chance node outcome was taken as a starting point for the next level of the game. The results with lower probabilities and their subsequent branches were excluded from the game tree. For clarity, the following game trees show some of these excluded branches as dotted lines. As a further limitation, only the weighted values of Wbk=0,01, Wbl=0,09 and Wrk=0,09, Wrl=0,01 were used. The W# was solved only by using a constant value of 0,9. This is to say that the blue player was risk averse and the red player was risk loving. This was thought to describe the natural preferences of the attacker and the defender. A deeper analysis would have required testing of different risk attitudes, risk behaviours and their impacts on expected utilities.

At the first level of the game tree both players selected their tactics from the available strategy spaces {LO,hi}. As the blue player was risk averse and the red player was risk loving their utility functions were;

Ub (bk, bl, b#)= 0,01*Ubk(bk)+0,09*Ubl(bl)+0,9Ubk(blpod#10)

Ur (rk, rl, r#)= 0,09*Urk(rk)+0,01*Url(rl)+0,9Ubk (rlpod#5)

And as discussed earlier;

0 bk ->Ubk 0, 4 bk->Ubk 1 and 0 bl->Ubl1, 4 bl->Ubl 0

0 rk->Urk 0, 4 rk->Urk 1 and 0 rl->Url1, 4 rl->Url 0

Based on the simulation the three most probable outcomes from the first level chance node $C_1\{B1_{LO}, R1_{hi}\}$ were u=0,66(3,1); u=0,33 (3,2); and u=0,11(2,2). Therefore, the overall utilities for the first level chance node $C_1\{B1_{LO}, R1_{hi}\}$ were;

a) p= 0,66

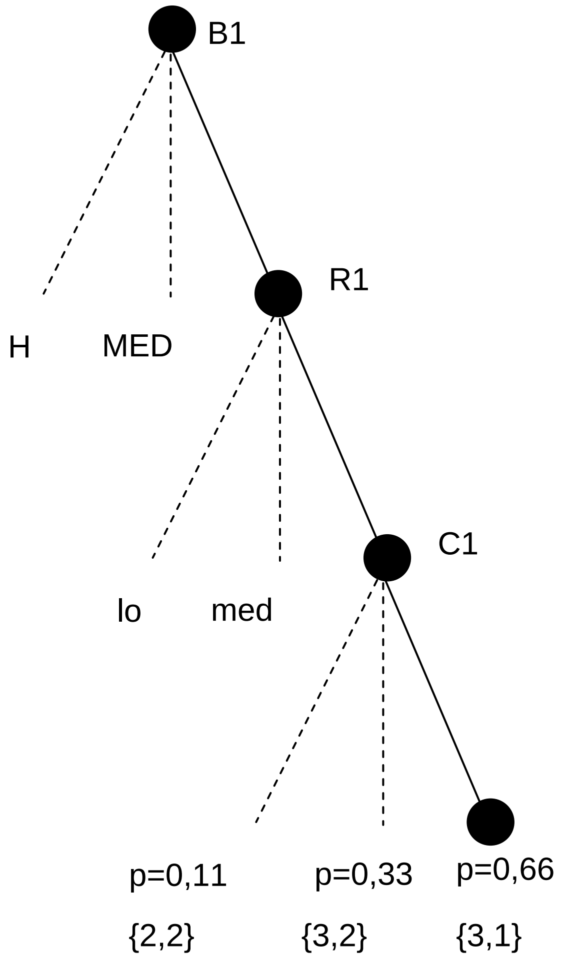
Ub (3,1)= 0,01*0,75+0,09*0,75+0,9*0= 0,0750
Ur (1,3)= 0,09*0,25+0,01*0,25+0,9*0= 0,0250

b) p=0,33

Ub (3,2)= 0,01*0,75+0,09*0,5+0,9*0= 0,0525
Ur (2,3)= 0,09*0,5+0,01*0,25+ 0,9*0= 0,0475

c) p=0,11

Ub (2,2)= 0,01*0,5+0,09*0,5+0,9*0= 0,0500
Ur (2,2)= 0,09*0,5+0,01*0,5+ 0,9*0= 0,0500



As seen above, the single branch of the game had expanded into three new starting points for the future sub-games. It is worth noting that both players received a value of 0,0 for the W#. This was due to the fact that after the simulation runs both the pod#5 and the pod#10 were still alive. Also, as there were both red and blue pods still remaining it was impossible at this stage to determine whether the blue was later able to kill the pod#10 or not and whether the red was later able to kill the pod#5 or not. Once the overall utilities were defined the expected utilities (EUb and EUr) were then calculated as seen below;

a) p=0,66

Eub (3,1)= 0, 0750* 0,66= 0,0495
Eur (1,3)= 0, 0250*0,66= 0,0165

b) p=0,33

Eub (3,2)= 0, 0525* 0,33= 0,0173

Eur (2,3)= 0, 0475*0,33= 0,0157

d) p=0,11

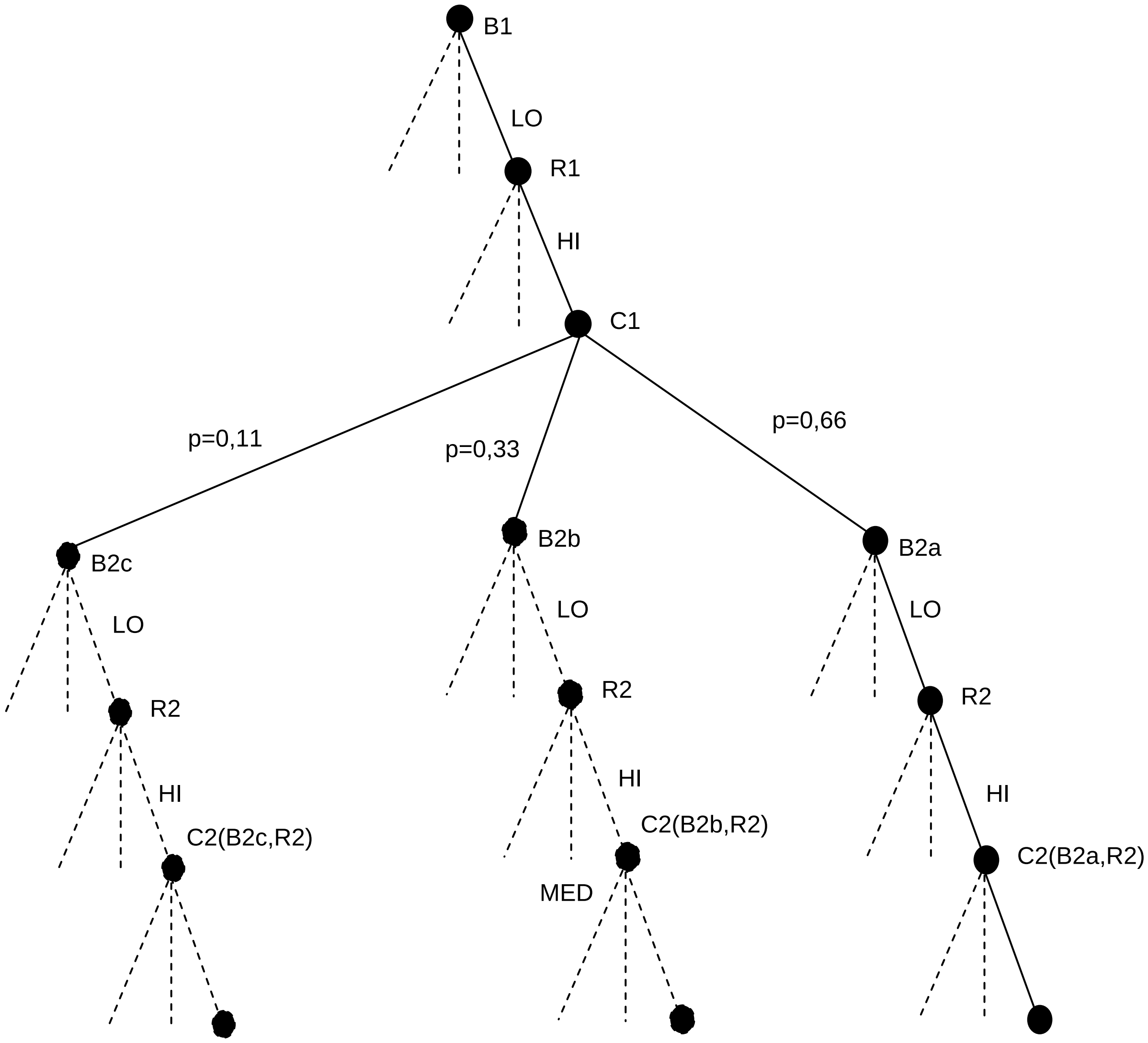
Eub (2,2)= 0, 0500* 0,11= 0,0550
Eur (2,2)= 0, 0500*0,11= 0,0550

Finally, these expected utilities were combined to present the player's overall expected utilities;

EUb{LO,hi}=0,66*Ub(3,1)+0,33*Ub(3,2)+0,11*Ub(2,2), which resulted to;

Eub {LO, hi}= 0,66*0,0750+0,33*0,0525+0,11*0,0500= 0,0723
Eur {hi, LO}= 0,66*0,0165+0,33*0,0157+0,11*0,0550= 0,0223

Depending on the selected branch of probability the player B could had now been in three different nodes. Therefore, the next level of the game tree had player B's decision nods B2a, B2b and B2c reflecting probabilities 0,66, 0,33 and 0,11 respectively. This part of the game tree is shown in the following picture. On a second level of the game tree the player B once again selected his tactic from the LO-strategy space and the player R selected his tactic from the hi-strategy space. It is worth emphasising that instead of running just one simulation at level C_2 the full solution of the level would have required a lot more simulations including cases such as; C_2 {B2a, R2}, C_2 {B2b, R2}, C_2 {B2c, R2}, those evolving from other strategy spaces than {LO,hi} and those resulting from different individual preferences towards risk. As mentioned earlier, for the purposes of this paper only the C_2 {B2a, R2} with p=0,66 was accepted to the next level of the game.



After completing the simulation using 100 monte carlo runs the ACEM-program produced the following results and probabilities for the second level of the game; C_2 ; C_2{B2a, R2} u=0,98(4,1), u=0,11 (4,2) and u=0,01(3,2). Based on those results the overall utilities for the second level chance node {B2a, R2} were;

a) p= 0,98

Ub (4,1)= 0,01*1+0,09*0,75+0,9*1= 0,9775
Ur (1,4)= 0,09*0,25+0,01*0,0+0,9*0= 0,0225

b) p=0,11

Ub (4,2)= 0,01*1+0,09*0,5+0,9*1= 0,955
Ur (2,4)= 0,09*0,5+0,01*0,0+ 0,9*0= 0,045

c) p=0,01

Ub (3,2)= 0,01*0,75+0,09*0,5+0,9*0= 0,0525
Ur (2,3)= 0,09*0,5+0,01*0,25+ 0,9*0= 0,0475

The expected utilities (EUb and EUr) were calculated for C_2 {B2a, R2} as seen below;

a) p=0,98

Eub (4,1)= 0, 9775* 0,98= 0,95795
Eur (4,2)= 0, 0225*0,98= 0,02205

b) p=0,11

Eub (4,2)= 0, 955* 0,11= 0,10505
Eur (2,4)= 0, 045*0,11= 0,00495

d) p=0,01

Eub (3,2)= 0, 0525* 0,01= 0,000525
Eur (2,3)= 0, 0475*0,01= 0,000475

After this the players' expected overall utilities for C_2 ; C_2{B2a, R2} were calculated as seen below;

EUb{LO,hi}=0,98*Ub(4,1)+0,11*Ub(4,2)+0,01*Ub(3,2)=
EUb{LO,hi}=0,98*0,95795+0,11*0,10505+0,01*0,000525= 0,95035175

EUr{hi,LO}=0,98*Ur(1,4)+0,11*Ur(2,4)+0,01*Ur(2,3)=
EUr{hi,LO}=0,98*0,02205+0,11*0,00495+0,01*0,000475=0,02215825

After the simulations at C_2 chance nodes, there was only a 1% chance that the game would evolve into the third level. When the third level simulations were processed, it resulted in Ub (4,0) with a probability of 1.00. Therefore the players' utilities for C_3 were;

Ub{LO,hi}=1,00*(4,0)= 1,00*1= 1

Ur{hi,LO}=1,00*Ur(0,4)= 1,00*0=0

At this stage this single branch of the game tree was considered to be completely solved. For the whole game tree to be complete, the process described here should have been iterated for game tree's all possible branches. After reaching all the possible leaf nodes and their expected overall utilities the game tree would have been completely solved. The solution for the game would then be found using the backward induction as described earlier in this paper. The game's solution shall not be discussed in greater detail as there is no complete game tree to be used for backward induction.

In conclusion, the pros and cons of game theory and more specifically those of the game formulated earlier are as follows. Gaining a general understanding of the game theory requires mainly logical skills and not that much mathematical understanding. But this applies only to the very basics of the theory. Once one starts to scratch below the surface and looks more closely at the nuances of the theory the topic rapidly gets extremely complicated. The formulation of an actual game, the building of the game trees and the generation of the required formulas all fall to the latter category. This paper argues that in order to enable the game theory to be effectively used in support of everyday military intuition, a fair amount of specialist skills will be required. But at the same time it must be said that it is unlikely that the game theory specialists could solve a complex military decision making problem in isolation. That is to say that in every game there are numerous case specific inputs and variables to be controlled and managed - all of which require deep understanding of the subject area for them to be interpreted correctly.[72] As a result, this paper argues that the game theory is not something that every military decision maker should adopt. Instead, understanding the way it approaches a problem in a form of a game and how it conceptualises even the most difficult problems, should be helpful for any decision maker at every level.

Even if the game would have a solid theoretical foundation there is still a good chance to get the results all wrong. While the theoretical principles remain the same from one game to another, the dynamics inside the games are usually somewhat tailored and unique. Those constructing a game have to make the critical decisions about the logic of the game. For example, the generation of reliable formulas and truthful weights within them is crucial. Games involving human interactions and human perceptions are generally highly difficult to define in a form of formulas and weights. This paper is suspicious to what extent soldier's, insurgent's or terrorist's risk behaviour or obedience to his or her superior commander can be transformed into a numeral format. As with in any analytical tool that is based on logical principles the game is just as good as the inputs and controlling rules inside it. The weights in this unclassified paper are examples only and they are not

---

[72] Virtanen, Kai, *Optimal Pilot Decisions and Flight Trajectories in Air Combat*, (Helsinki: University of Technology Systems Analysis Laboratory), Research Reports A90, 2005, p. 10

even trying to represent reality. They are only to clarify the game's mechanics and would in real life be replaced by some other more representative values. It is left for the readers to judge, whether the logic of the equations is correct in the first place and whether there are representative values to be found at all.

Games based on game theory assume players to be rational. This premise provides the foundation for other assumptions and is a cornerstone for the logical game dynamics. This paper basically shares the game theory's view about the rationality. That being said, it has to be elaborated that the rationale as a behavioural control phenomenon is difficult to measure and interpret. Rationality has to be defined through cultural and individual perceptions and values. There lies a risk that a game is formulated using biased perceptions, namely those represented by the ones constructing the game. In a military context, it could be a difficult task for a western game theorist or military decision maker to capture the personal rationales of a suicide bomber or freedom fighters fighting against an seemingly overwhelming enemy. As a result, this paper argues that without a deep cultural understanding any game will most likely result in incorrect results. For example, in the game formulated in this paper it is highly questionable to claim the solution as an absolute truth. If all the inputs and parameters (excluding the comprehensive cultural understanding) were correct, only the minimum losses suffered by the enemy could be reliably measured. Own losses would be more hard to predict as the enemy might understand the principles of rationale differently than we. The enemy might not select a strategy combination that results in maximum utility, but the one that simply minimises the blue player's utility.

The game theory makes several strict assumptions about the information available to players. As shown earlier, players may have understanding about the previous moves and they may know in what branch of the game tree they currently are at. Also, players are assumed to have knowledge of the expected outcomes of future moves, thereby helping them to make rational choices. In interactive decision making situations the information available may be more limited than the principles of the game theory suggests. A player may be extremely limited in his situational awareness. He may not know what has happened earlier, he may not precisely know the branch he is at and most likely he has no complete mental picture about what are the long term consequences of his short term choices. To understand this better, one should remember that it is possible that a player's choice resulting in less than optimal immediate outcome at some early stage of the game may actually be the choice resulting in the most optimal leaf node. Even if a player's rationale was interpreted correctly, his lack of situational awareness might result in a behaviour that seems less than rational. This shows just how essential it is to understand the amount and quality of the information available to the players.

The game tree is an essential part of the game. It provides a framework over which the game unfolds itself. It is more than a tool to visualise the game; it is a structure that captures the whole dynamics of the game. Even if the game tree would not be used for actual game solving, it provides an excellent tool to conceptualise a difficult decision making problem. If it is hard to capture a problem into a form of a game tree, it is probably even to harder to conceptualise the problem without it. For the actual problem solving the game theory only takes few additional steps from the simple visualisation. It adds the players' actual outcomes and shows clearly which are the decisions optimising each actor's individual outcomes. Also, a game tree combined with a backward induction enables the optimum strategy combination to be defined. The backward induction is a rather simple method once the game tree is finished with all the chance node outcomes and the expected utilities in place. Unfortunately, the game tree may easily become exhaustingly large. It must be emphasised that in this paper the example game had only three possible behavioural options out of which two were completely left out for simplicity. Even then the probability based alternative realities and subsequent levels and branches made the game fairly excessive. The use of different risk levels and risk attitudes would each have multiplied the size of the game tree. It must be acknowledged that the mere size of the game tree may reduce its usefulness if it is used as a demonstration tool rather than an actual game theoretic problem solving method.

A simulation tool that supports and partially solves game trees is another source of inaccuracies. In a sense the simulation tools are much like the game trees. They both follow specified logical and capability related rules which are based on intelligence, testing and guessing. These hidden inaccuracies inside the simulations are often forgotten in a same way as they are forgotten in games. Even more, they easily cumulate with those inaccuracies of the game. If the game is to be kept simple, only rough values are to be used. This leads to results that are not representative. And if the number of players and variables is increased, the results are probably more representative as more factors are being taken into consideration. But at the same time, the number of possible sources of inaccuracies increases.

The game described in this paper is basically a simplified demonstration of a simple problem showing how to use the game theoretic principles for its solving. But as games are very cost effective method of testing any interaction they are used for more difficult scenarios too. The more there are risks and costs involved in the decision making situation the more likely it is that the principles of game theory are being used to solve it. And military decisions are not an exception as whole military campaigns and operations have been and will be tested using games. But can the game theory be used to enhance military intuition? Considering all the aspects discussed here, the answer has to be 'yes'. Game theory may not be the optimal tool to turn a complex decision making problem into a numerical form. But the expected success in military campaign needs to be measured somehow and for the time being the game theory still provides the most reliable approach for that purpose.

For an ordinary warfighter the game theory may provide an analytical framework through which simple problems could be approached. For them the game theory could be at its best when used as a learning tool rather than as an actual method for providing absolute answers in a decision making process. Using the principles described in this paper the military researchers could improve their understanding about the typical military problems by testing how different variables impact the outcome. A game theory could help them in creating an understanding about the impact of tolerances and about the risks involved in different choices. If nothing else, an understanding of the principles of game theory helps one to see how complex the war can be and how false a badly constructed analysis can be. All these together should help a military decision maker at any level to enhance his intuition, at least indirectly.

Bibliography

Books

Anthony, Kelly, ed., *Decision Making Using Game Theory: An Introduction for Managers* (New York: Cambridge University Press, 2003).

Carmichael, Fiona, *A Guide to Game Theory*, (Harlow, Essex: Prentice Hall, 2005).

Clemen, Robert T, *Making Hard Decisions, An Introduction to Decision Analysis*, (Duke: Duxbury Press,1996).

Coram, Alex T., *State, Anarchy and Collective Decisions* (New York: Palgrave, 2000).

Gale, Douglas, *Strategic Foundations of General Equilibrium: Dynamic Matching and Bargaining Games*, (Cambridge, England: Cambridge University Press, 2000).

Gibbons, R., *A Primer in Game Theory*, (Harlow, Essex: Prentice Hall,1992).

Kreps, David, M., *Game Theory and Economic Modelling,* (Oxford: Clarendon Press, 1990).

Rapoport, A*, Fights, Games, and Debates* (Ann Arbor, Michigan: University of Michigan Press, 1960).

Rasmusen, E., *Games and Information: An Introduction to Game Theory* (Oxford, Blackwell Publishers, 2001).

Shubik,M., *Readings in Game Theory and Political Behavior* (Garden City, New York: Doubleday, 1954).

Vega-Redondo, F., *Economics and the Theory of Games* (Cambridge, England: Cambridge University Press, 2003).

Articles & Publications

ACEM operator's guide, version 4.5F, (Raytheon Missile Systems, Operations Research Department), 2003.

LtCol Cantwell L., Gregory, *Can two person zero sum game theory improve military decision making course of action selection?*, (Fort Leavenworth, Kansas: School of Advanced Military Studies, United States Army Command and General Staff College, 2003), A Monograph.

Karelahti, Janne, *Untitled*, (Helsinki: University of Technology, System Analysis Laboratory, 2006).

Karelahti, J, Virtanen, K and Raivio,T, *Game Optimal Support Time of a Medium Range Air-to-Air Missile*, (Helsinki: University of Technology Systems Analysis Laboratory), Research Reports, 2004.

Ministry of Defence, The Joint Doctrine and Concepts Center, *Joint Warfare Publication 5-00 Joint Operations Planning* (Ministry of Defence, 2004), Annex A.

Poropudas, Jirka, *Ilmataistelun simulointimallin peliteoreettinen analyysi* (Helsinki: University of Technology,System analysis laboratory, 2005).

Virtanen, Kai, *Optimal Pilot Decisions and Flight Trajectories in Air Combat*, (Helsinki: University of Technology Systems Analysis Laboratory), Research Reports A90, 2005.

Virtanen, K., Raivio, T., and Hämäläinen, R. P., ’*Decision theoretical approach to pilot simulation*’, Journal of Aircraft 36, number 4,paper 0021-8669, Universal Technology Corporation, 1999.

Internet Sources

Budescu, David V., Erev Ido, and Zwick, Rami, eds., Games and Human Behavior: Essays in Honor of Amnon Rapoport (Mahwah, NJ: Lawrence Erlbaum Associates, 1999), [online], Available from: http://www.questia.com/PM.qst?a=o&d=26304023, [Accessed 2 May 2007].

Gale, Douglas, Strategic Foundations of General Equilibrium: Dynamic Matching and Bargaining Games (Cambridge, England: Cambridge University Press, 2000), [online], Available from: http://www.questia.com/PM.qst?a=o&d=105724281, [Accessed 15 April 2007].

Graham, Edward M., Market Structure and the Multinational Enterprise: A Game-Theoretic Approach, Journal of International Business Studies 29.1 (1998), [online], Available from: http://www.questia.com/PM.qst?a=o&d=5001338111, [Accessed 19 May 2007].

Harris, Andrew C., and Madden Gregory J., Delay Discounting and Performance on the Prisoner's Dilemma Game, The Psychological Record 52.4 (2002), [online], Available from: http://www.questia.com/PM.qst?a=o&d=5000677784, [Accessed 19 May 2007].

Linster, Bruce G., Evolutionary Stability in the Infinitely Repeated Prisoners’ Dilemma Played by Two-State Moore Machines, Southern Economic Journal 58.4 (1992),[online], Available from: http://www.questia.com/PM.qst?a=o&d=5000143424, [Accessed 28 March 2007].

Rullière, Jean-Louis and Walliser, Bernard, 7 From Specularity to Temporality in Game Theory, Game Theory and Economic Analysis, ed. (London: Routledge, 2002), [online], Available from: http://www.questia.com/PM.qst?a=o&d=103310867, [Accessed 30. March 2007].

Stearns, Maxwell L., A Beautiful Mend: A Game Theoretical Analysis of the Dormant Commerce Clause Doctrine, William and Mary Law Review 45.1, 19 May (2007), [online], Available from: http://www.questia.com/PM.qst?a=o&d=5002058582, [Accessed 19 May 2007].